Facultad Latinoamericana de Ciencias Sociales, FLACSO Ecuador
Departamento de Desarrollo, Ambiente y Territorio
Convocatoria 2015-2018

Tesis para obtener el título de doctorado en Economía del Desarrollo

Networks: Empirical and Theoretical analysis

José Fernando Ramírez Álvarez

Advisor: Wilson Pérez Oviedo
Co-advisor: Juan Ponce
Doctoral Committee: Leonardo Vera, Fernando Martin, Juan Fernandez, Paul Carrillo and Victor Aguiar

Quito, September 2019

Dedication

To my beloved wife, Alexandra Serrano Flores

Table of Contents

Illustrations

Declaration of assignment of right of publication of the thesis

I José Fernando Ramírez Álvarez, author of the thesis "Networks: Empirical and Theorical analysis", I declare that this research is my exclusive authorship, that I have prepared it in order to obtain the Doctor in development economics grade by Facultad Latinoamericana de Ciencias Sociales, FLACSO Ecuador.

I give rights exclusive rights of reproduction, public communication, distribution and dissemination to FLACSO Ecuador, under the Creative Commons 3.0 Ecuador license, for this university to publish in its institutional repository, as long as the objective is not to obtain an economic benefit.

Quito, September 2019

José Fernando Ramírez Álvarez

Abstract

In the last two decades, economic analysis has gradually incorporated the network theory, broadening the standard microeconomic models by considering dependent agents in contexts of: game theory, social capital, labor and financial markets, productive networks and experiments. This thesis investigates two big topics about networks in economy: the interaction between accountants and taxpayers in firm's tax reporting, and the diffusion of microeconomic shocks through input-output interrelations. The former is an empiric investigation that was carried out through an experiment for deterrent notifications on Ecuadorian tax system in 2016. Meanwhile, the latter is a theoretical investigation that analyzes the role of imperfect markets and firm bankruptcy through a novel microeconomic network model.

The results show interesting evidences about the importance of networks in tax fulfillment and shock diffusion. For the first topic, it was found that deterrent electronic notifications not only over taxpayers but also over their accountants could improve firms' tax reporting. Mainly, the strongest effect on firms' declared tax was obtained when both accountants and taxpayers were notified simultaneously with reciprocal awareness (i.e. each one knows the other was notified). This finding a systemic relationship between both parties through an agency problem, as it is stated in tax literature.

For the second topic, it was found that a microeconomic shock in productive networks with price fixing and firm bankruptcy could generate different kinds of cascade effects on prices and quantities, complementing what the literature on shock diffusion points out when perfect competition is assumed. In particular, there is an upstream cascade effect on firm bankruptcy when a simple non-complex productive network is analyzed. Here, a productive shock on a firm increases its selling prices, reduces its possibility of trading in the network and decreases its expected profits. Hence, its probability of bankruptcy increases, causing the activity of its regular suppliers, the regular suppliers of these suppliers, and so on, worsen. If some firm of this regular suppliers' chain goes bankrupt in any period, the rest of the firms will have a higher probability of bankruptcy in the next periods, thus reproducing and intensifying the effect over time.

Introduction

This thesis investigates two topics about networks in economy: the interaction between accountants and taxpayers in firm’s tax reporting when any of them (or both) are notified by tax administration, and the diffusion of microeconomic shocks through input-output interrelations in economies with imperfect competition and firms’ bankruptcy.

Why do these issues are they important to study? On the one side, accountants can be considered key agents in the firms’ social capital network due to their legal liability in firms’ tax reporting, their knowledge on accounting rules and tax evasion practices and the fact that they can work for several firms. Here, some microeconomic theories explain how these individuals interact with taxpayers in corporate tax evasion through an agency problem, however, to date there is still no empirical evidence to show it. On the other side, the productive network is an important economic mechanism to understand how microeconomic shocks propagate in the economic system and generate macroeconomic fluctuations. In this area, several empirical and theoretical researches have assumed perfect competition, without rivalry and firm exit. These omissions can distort the diffusion of a microeconomic shock in the productive network, lessen its propagation or limit it to a certain group of firms.

Even though these topics have a common background (i.e. networks), they have a different scope and methodology, so it is recommended to read them separately in the chapters described below.

The first chapter analyzes the role that accountants play in corporate income tax evasion in Ecuador. For this purpose, an experiment was carried out in Ecuadorian tax system with microenterprises in early 2016. The experiment evaluates to what extent a persuasive message on accountants is more effective than on taxpayers, through five different notifications. Three notifications focused only on accountants. Here there were a placebo, a notification with penalty message (years of imprisonment), and a notification with accountants’ private information (the number of firms the accountant work for). The fourth notification focused on both accountants and taxpayers, displayed accountants’ private information and stressed the reciprocal awareness about the notification (i.e. each one knew the other was notified). The fifth notification focused only on taxpayers and displayed a penalty message. All these notifications were electronically sent via tax box before tax reporting deadlines.

Using a simple regression model, the results show that simultaneous notifications on both accountants and taxpayers were the only treatment that increased significantly firms' declared income tax. They were even more effective at improving firms' declared tax than notifications on accountants only. Furthermore, it was shown that penalty notifications on accountants, rather than taxpayers only, were the most significant treatment at increasing firms' declared revenue, however they did not generate a significant impact on declared tax due to a cost overreporting mechanism.

The second chapter studies the diffusion of productivity shocks in networks of firms, where there is imperfect competition and firms' bankruptcy is a possibility. For imperfect competition, it is assumed that each firm has to choose, for each one of its inputs, a supplier among a subset of competing firms. When it does so, the buyer accepts the supplier's selling price and can demand the amount of the input it requires, however, the transaction could fail with some probability (for example, if goods do not have the expected quality) causing losses to the buyer. This probability is greater for non-regular suppliers, who have to offer lower prices in order to compete. The buyer decides not only looking for the highest expected profit, but also for the lowest risk of loss that each supplier offers, leading him to assign a probability of hiring to each bidder. Hence, the buyer's decision is modeled differently from the classic maximization of expected utility, allowing us to represent a continuum of market power, different from the classic Cournot or Bertrand.

Regarding firm bankruptcy, once the assignments of contracts are made, all the goods are produced, wages are paid and the representative household defines its consumption, the profits of firms are realized. Those firms with worst relative performance for longer time have greater possibilities of bankruptcy. The existence of the equilibrium in such an economy, which includes a Nash Equilibrium with prices as strategies and supply-demand equilibrium, is shown.

The simulations of the model for a simple non-complex productive network show different cascade effects on the prices and economic flows when a firm gets a shock. First, there are two types of price effects in the initial period: a horizontal effect and a downstream cascade effect. Second, there is an upstream effect on firms' profit in the initial period. Finally, there is an upstream cascade effect on firms' bankruptcy that is propagated exclusively to regular suppliers in the network over time.

Chapter 1

The Accountant as a means to corporate tax evasion: Evidence from a field experiment[*]

1.1 Introduction

Tax evasion is considered one of the biggest problem for development in the world (Tax Justice Network, 2007). Its high cost to society has continuously forced researchers and tax professionals to rethink a way to improve tax enforcement policies.[1] As Murphy and Christensen (2013), tax evasion could be understood as a form of organized crime, since it includes several parties such as taxpayers, lawyers, banks and multinational entities. These parties interact with one another through a complex economic, financial and institutional network, in order to forego tax obligations and increase profits.

In this process, accountants are a key agent due to their legal liability in a firms' tax reporting and the privileged information they have on accounting rules, permissible deductions and exempt revenues to fulfil tax obligations and reduce tax burdens.

This role is more visible when a single accountant is able to work for several firms, each of which may implement compensation mechanisms to prompt efficient tax decisions and mitigate the risk of detection for wrongdoing. To date, there is a lot of empirical evidence about accountant's incentives for tax avoidance, (e.g., Armstrong, Blouin, and Larcker 2012; Gaertner 2014; Powers, Robinson, and Stomberg 2016) however, there are few theoretical studies that focus on accountants' behavior inside firm (Crocker and Slemrod 2005; Chen and Chu 2005; M. Desai, Dyck, and Zingales 2007; Biswas, Marchese, and Privileggi 2013) without any empirical causal test of tax enforcement policies.

The aim of this paper is to analyze the role that accountants have and their interactions within firm's tax evasion through an experiment carried out in Ecuador's tax system at the beginning

[*] I thank Guillermo de la Cruces, Ricardo Truglia, Hessel Osrterbeek, Juan Ponce and Paul Carrillo for helpful comments and discussions. I am grateful to the staff at the Ecuadorian Tax Authority (Servicio de Rentas Internas) for their active collaboration and for providing access to the data used in this study. I am also grateful to Carlos Jaramillo, and José Almeida, and many others.

[1] Based on several studies, Tax Justice Network (2007) estimates that less-developing countries evade close to USD 200 billion in taxes each year, by personal offshore practices, corporate profit transfers and informality. To get an idea, this fiscal loss is so high that doubles the budget established within the Millennium Development Goals to halve world poverty in a decade.

of 2016. The experiment sent five different kinds of electronic deterrent notifications to accountants and taxpayers prior to the reporting deadline. The universe of the experiment was composed exclusively by microenterprises. The first three notifications focused only on accountants. Here there were a placebo, a notification with penalty message (years of imprisonment.in case of tax evasion), and a notification with accountants' private information (the number of firms the accountant keeps accounts for). The fourth notification focused on both accountants and taxpayers, displayed accountants' private information and stressed the reciprocal awareness of both parties (i.e. each one knew the other was notified). The fifth notification focused only on taxpayers and displayed a penalty message. These treatments allowed to evaluate the effect of accountants' notifications on firm's tax reporting and under what conditions this effect happen.

For the most part, the results show that simultaneous notifications on both accountants and taxpayers were the unique treatment that increased significantly firms' declared income tax. They were even more effective at improving firms' declared tax than notifications on accountants only. This suggests a possible interaction between accountants and taxpayers when both are notified, which increases the risk of detection, involves a better tax compliance and improves tax reporting.

Furthermore, it was shown that penalty notifications on accountants, rather than taxpayers only, were the most significant treatment at increasing firms' declared revenue. However, this effect did not transfer to declared tax since it was found a cost overreporting mechanism that canceled any effective tax increase. A possible reason for this could be the standard message that the tax administration provided in notifications, which mitigate the risk perceived by taxpayers.

Interestingly, simultaneous notifications on both parties demonstrated to be more effective for firms whose revenues and costs are higher, when interaction terms are included. These notifications demonstrated also be more effective for firms whose accountants work for several companies, and for firms whose accountants are young, but with lower statistical significance.

This study provides initial insight about the effect of deterrent policies addressed to accountants obtained in the field. Within literature about the evaluation of tax enforcement policies, to date there is no experimental study concerning how a deterrent policy could affect the accountants' behavior and hence the firms' tax reporting. As such, this study provides a unique contribution to tax research literature as most field studies focus on taxpayers instead of other economic agents that interact with them.

In addition, this study provides initial empirical evidence to microeconomic theories on the interaction between an accountant and a firm owner in tax reporting. Although Crocker and Slemrod (2005), Chen and Chu (2005), and Biswas, Marchese, and Privileggi (2013) provides guidelines relating to accountant's participation in tax evasion through an agency problem and contract theory, so far there is no empirical testing of them. Lastly, this study contributes to the increasing but limited literature on tax compliance through field experiments (e.g. Hasseldine et al. 2007; Appelgren 2008; Iyer, Reckers, and Sanders 2010; Kleven et al. 2011; Ariel 2012; Pomeranz 2013; Harju, Kosonen, and Ropponen 2014; Kosonen and Ropponen 2015; Carrillo, Pomeranz, and Singhal 2017).

The remainder of this paper is structured as follows: Section 2 provides the main theoretical approaches for understanding the tax evasion problem and highlights the most relevant empirical evidence based on experiments. Section 3 describes the experimental design that was used for this study and shows some data statistics about the experiment's universe. Section 4 analizes the main results found for revenue reporting, cost reporting and tax reporting through a econometric model. Section 5 provides conclusions.

1.2 Literature review

This literature review has two main purposes. Firstly, it looks to introduce the microeconomic theory on firm's tax evasion with emphasis in agency models. Secondly, it looks to present empirical evidence found in previous experiments regarding the effect of deterrent and non-deterrent policies. This part places special focus on field experiments on firms' tax fulfilment.

1.2.1 Theoretical approach

The theoretical analysis of tax evasion has its origin in the model proposed by Allingham and Sandmo (1972), also known as the AS model. [2] This model represents an extension of the

[2] This model is also known as TAG model (Taxpayer as Gambler) (Cowell 2004).

crime model elaborated by Becker (1968), which studies optimal policy settings (civil damages, arrest costs, conviction, prison sentence, fines, and so on) in order to prevent illegal activities.

The AS model is a microeconomic model based on choice under risk, where a risk-averse individual decides to hide his income to evade taxes and maximize expected utility. This occurs under a set of exogenous parameters such as the probability of being caught (also denoted as tax monitoring), the penalty or fine, the tax rate, and the income generation. This problem involves two situations: not being detected, which means an additional income equivalent to the amount evaded; or being detected, which generates a fine payment.

Based on this model, Allingham and Sandmo (1972) show that an individual will evade income tax when the expected return per unit of income is higher than the expected cost of hiding it. In this case, tax evasion is more likely to occur when: i) the probability of getting caught decreases, ii) the penalty decreases, iii) the tax rate increases, iv) the risk aversion decreases and, v) income is higher (Cowell 2004).

The AS model has laid the basis for several theoretical analyzes on tax evasion, with further implications on labor supply, optimal taxation, uncertainty, informal markets, imperfect information, interaction with tax administration, design mechanisms, moral issues, and social dynamics.[3] A topic that has gained interest in recent years is the behavior of firms when it comes to tax evasion.

Most theory on firm's tax evasion has adopted an individual approach of the AS model considering endogenous income. Here, income is generated by profit maximization, rather than by individual preference maximization with leisure and labor supply. For the most part, the main research interest in this area has been the separability between production and tax evasion decisions. In other words, under what conditions unreported income does not depend on production levels, and such production levels do not depend on tax monitoring or the penalty established by the tax administration (e.g., Marrelli 1984; Kreutzer and Lee 1988; Virmani 1989). Currently, academics are also interested in the industrial transactional

[3] Andreoni, Erard, and Feinstein (1998), Myles (2001), Slemrod and Yitzhaki (2002), Cowell (2004), Sandmo (2005) and Torgler (2007) make a quick description about a great deal of this research.

framework and how it might influence the firms' behavior through third-party information (e.g. Kopczuk and Slemrod 2006; Gordon and Li 2009; Kleven, Kreiner, and Saez 2016; Carrillo, Pomeranz, and Singhal 2017).

The firm's tax evasion problem under the individual approach of the AS model has a disadvantage. It does not look at how the accountant/tax manager/CFO and how the owner/shareholders participate and interact when it comes to the decision-making process of a company. On one side, company owners have a vested interest in increasing profits by means of tax evasion, while on the other side, accountants have the legal (and illegal) knowhow to make it possible. Slemrod (2004) considers that the compensation mechanisms between these two parties, either by formal or informal means, are key features to understand the effects of tax deterrent policies, especially in large companies.

This theoretical limitation has led to the re-thinking firm's tax evasion as an agency problem, where the roles of company owners and accountants differ accordingly. This formulation involves a compensation mechanism through a contractual relationship that mitigates the risk assumed by accountants when evading taxes on behalf of firms. In other words, this mechanism encourages accountants to make efficient tax decisions that can decrease firm's tax payment. Chen and Chu (2005), Slemrod and Crocker (2005) y Biswas, Marchese, and Privileggi (2013) are main references to this point.

Chen and Chu (2005) argue that tax evasion is essentially determined by the trade-off between the expected return, the risk of being caught, and a company's management control. They analyze tax evasion through income under-reporting, assuming there is an incomplete contractual relationship[4] between a company owner and a risk-averse tax manager. When both parties are legally liable for tax fulfillment, they find that tax evasion generates a loss of internal control and inefficiency at a company. The reason for this result is simple: contract incompleteness does not fully mitigate the risk taken by tax managers given that their compensation may be less than it would have been without evasion. This fact discourages managers from giving their best effort, generating agency costs for the firm they work for.

[4] An incomplete contract is a kind of contract that compensates accountants with an ex-ante risk-premium (before fraud happens). This contract must not be specific regarding the terms of compensation since tax evasion is an illegal activity.

Crocker and Slemrod (2005) developed a model with a similar theoretical approach. They examine the optimal contractual relationship that should occur between company owners and tax managers (i.e. accountants) when both are risk-neutral, assuming that the latter possesses privileged information on permissible legal deductions which might be over-reported in order to reduce corporate income tax (in principle, this information is unknown for owners, i.e. there is information asymmetry). Thus, the authors find that tax penalties on tax managers are more effective in reducing evasion than those on shareholders if both parties are legally liable for tax fulfillment.[5] This result is essentially due to the information asymmetry embedded in tax reporting and the possibility that accountants can be also penalized when firms are caught by tax administration. These aspects could weaken the compensation managers should receive for reducing risk and exacerbate the conflict between the two parties when penalties on accountants increase.[6]

Another interesting contribution in firm's tax evasion through agency models is made by Biswas, Marchese, and Privileggi (2013), who assumed that tax managers can carry out self-protective actions to under-report firm's income without increasing the risk of being caught. Here, the probability of being caught is endogenous and adversely depends on tax manager's effort. Under these considerations, the authors show that not only tax evasion but also tax manager's efforts may decrease when liability is gradually shifted from the principal to the agent. This is more likely to occur in situations where there are high tax penalties and high-risk levels of aversion. Evidently, highly risk-averse tax managers with greater liability properly report taxes to avoid penalties, however, if their compensation decreases after the liability shifts, the likelihood of properly reporting taxes will also decrease because the compensation includes an ex-ante premium that lessens the risk of being caught.

1.2.2 Empirical evidence

The strongest empirical evidence on the effects of tax enforcement policies has been found through experiments, either lab experiments or field experiments. These tools allow for the

---

[5] Properly, Crocker and Slemrod's model (2005) analyzes how the expected cost of incurring tax evasion affects deduction overreporting. As pointed out by the authors, this cost can increase not only by tax penalties, but also by other enforcement parameters such as the likelihood of being caught. So, the conclusion remains the same for policies that also change tax monitoring.

[6] According to Crocker and Slemrod (2005), the information asymmetry is similar to the incomplete contractual relationship specified by Chen and Chu (2005). The illegality of an incomplete contract stresses the penalty effect on tax managers, which makes them to demand a higher compensation to mitigate the risk of being caught.

determining of cause-effect relationships in individuals' behavior by randomly assigning different "treatments" and evaluating their impact on socio-economic variables. Essentially, such treatments are implemented through letters, pamphlets, informational videos, surveys or interviews with the purpose of increasing perception about tax monitoring, punishment or moral principles.

Lab experiments have been the most used tool to analyze tax evasion within little groups of individuals. Several studies show that taxpayer assistance, withholding taxes, the uncertainty about tax enforcement, the moral constraints, the public spending, the social perception of equity and the social interaction are fundamental aspects to determine how notifications influence tax fulfillment.[7] Although lab experiments have the advantage of incorporating several factors in the behavioral analysis of tax evasion (which in some cases can be very difficult to study using a theoretical microeconomic model), their results cannot be extrapolated to the population of tax systems due to their loss of representativeness and external validity (Levitt and List 2007). Field experiments remove this drawback and lead to representative results for a universe of taxpayers with real tax obligations.

Field experiments on tax evasion are less common in the literature than laboratory experiments. However, nowadays its use has intensified gradually thanks to tax administrations and their interest in improving their process.[8] These techniques generally involve sending either physical or electronic letters to a segment of high-risk taxpayers prior to the fulfillment of tax obligations. The information given is generally focused on deterrent issues (e.g., audit threats, penalty reminders), taxpayer assistance (e.g., service reminders, contact information), public services (e.g., public financing, redistributive issues), and social norms (e.g., morality, social stigma). In summary, results indicate that deterrent notifications usually upsurge fulfillment of tax obligations. A large compilation of field experiments on tax evasion and their main results can be found in Hallsworth (2014).

---

[7] Some examples of these sutides are Robben et al. (1990); Alm, Jackson, and McKee (1992); Fortin and Lacroix (2007); Alm et al. (2010); Coricelli, Rusconi, and Villeval (2014); Bazart and Bonein (2014).

[8] As expected, this type of research requires the cooperation of tax administration in many aspects such as access to tax files, policy or control mechanisms, interviews to tax experts and lawyers, availability of communication channels, among others. The first study of this class was the experiment series of Minesota's Revenue Department. Coleman (1996) and Slemrod, Blumenthal, and Christian (2001) makes a broad description about them.

In the context of firms, field experiments have been the most reliable means for empirical analysis of tax fulfillment. Firms' behavior must be observed in real situations due its complexities, interrelations and dynamics, so the evaluation of any tax enforcement policy requires be on-site. Field experiment studies here mainly focus on deterrent notifications about income tax, value-added tax and some subnational taxes. Most of these notifications are via email, focused on penalty and deadline reminders, and sometimes includes third-party information.

There are three key findings in field experiments for firm's tax evasion. The first one relates to the importance of paper trails and third-party information for improving tax reporting. For example, Kleven et al. (2011) elaborated a tax enforcement field experiment in Denmark that randomly audited and sent threat-audit letters to taxpayers. The authors found that these interventions had significant effects on self-reported income (i.e. income reported by taxpayers themselves), but no effects on third-party reported income (i.e. income reported by others taxpayers). This result occurs because third-party information increases the monitoring perception and the risk of being caught, so tax evasion for third-party reported income is extremely small prior any tax enforcement policy.

Secondly, there is a kind of tax evasion substitution between the declared revenue and the declared cost in tax return. Carrillo, Pomeranz, and Singhal (2017) supported this idea through a field experiment in Ecuador with electronic notifications. These notifications communicated to firms any income party-difference in their tax reporting with additional penalty reminder. As a result, authors found that declared revenue of notified firms largely increased, removing any income party-difference that has been revealed. However, declared costs also increase at a similar rate, generating a non-appreciable impact on income tax. In other words, tax evasion, through revenue underreporting, is substituted by tax evasion through cost overreporting.

The third main finding relates to the spillover effect of the economic productive network. Pomeranz (2013) designed an audit preannouncement experiment that targeted small firms in the Chilean tax system. The study shows that declared added-valued tax increases not only for the notified firms, but also for their trading partners. This propagation happens in a backwards manner within the productive network (i.e. from supplier to supplier), either suppliers overstate their sales (and consequently their tax payments) or they collude with the intervened firm to match their transactions.

Despite this valuable evidence, to date there is no research to explain how tax enforcement policies could affect accountants' behavior and their participation in firm's tax reporting. Furthermore, the agency models for firm's tax evasion problem lack field testing.[9] There is only limited empirical evidence on accountants' compensation incentives for tax evasion through cross-sectional correlation analysis (e.g., Phillips 2003; M. A. Desai and Dharmapala 2006; Hanlon and Heitzman 2009; Armstrong, Blouin, and Larcker 2012; Gaertner 2014; Powers, Robinson, and Stomberg 2016).

Among the theoretical and empirical findings previously discussed, there remain doubts concerning corporate income tax. For example, is the accountant a critical agent regarding taxpayers' social capital? What about tax enforcement policies on accountants? Is it more effective to increase tax monitoring on accountants, taxpayers or both? Are the results the same when there is a full-information environment between accountants and taxpayers? What happens if accountants provide accounting services not only to one but several firms? Are the effects the same on revenue, cost and tax reporting? These questions are answered through the impact evaluation shown below.

1.3 Experiment design, data and empirical specification

1.3.1 Empirical setting

According to the ecuadorian tax administration,[10] the country's tax revenue made up approximately 14% of GDP in 2015, excluding social security contributions and sectional taxes. Direct taxes represent approximately 36% of the tax burden and indirect taxes 60%. Within direct taxes, corporate income tax is the most important form of revenue. About nine out of every 10 dollars from direct tax collection come from companies' profits while the remainder is obtained from personal income tax and other taxes.

---

[9] Barile (2012) is perhaps the only reference to the date that evaluates Chen and Chu (2005)'s hypothesis about the relationship between efficiency of internal control and tax evasion. However, it is done by a laboratory experiment.

[10] The tax administration in Ecuador collects 12 types of taxes: corporate income tax, personal income tax, simplified Ecuadorian tax scheme, added value tax, special consumption taxes, foreign outflows tax, motorized vehicle tax, rural land tax, foreign asset tax, vehicular pollution tax, non-returnable plastic bottles tax and petroleum royalties.

Ecuador's tax evasion gap of corporate income tax ammounted to 63.5% in 2008 (Jiménez, Sabaini, and Podestá 2010), representing 6% of that year's GDP. Meanwhile, coporate tax expenditure (e.g. employment and investment stimuli) in 2015 represented approximately 1.5% of that year's GDP.

The reporting of corporate income tax is done annually through the F101 form. Through it, firms are required to declare their revenues, expenditures and pre-tax profits for their economic activities. In order to determine the tax base, 15% of worker profit participation is deducted together with several exemptions and deductions.[11] Consequently, income tax is determined by two flat rate which include 22% for regular profits and 12% for profits that wil be reinvested.

Corporate tax returns are filed electronically every April according to a reporting deadline, after economic activity was developed. A tax box is available for this purpose, through which taxpayers also receive communications about outstanding and deterrent tax issues.

1.3.2 Experiment design

In order to see how accountants and taxpayers participate and interact in firms' tax evasion practices, a field experiment was designed in collaboration with Ecuador's tax administration on corporate income tax reporting of fiscal year 2015. Each treatment of this experiment involved sending one of the following tax deterrent notifications:

- **T1**. Accountant placebo notification
  It reminds accountants of the deadline of income tax reporting.
- **T2**. Accountant penalty notification.
  In addition to the message in treatment T1, it reminds accountants of the penalties in case of tax evasion, mainly years of imprisonment.
- **T3**. Accountant risk notification
  In addition to the message in treatment T1, it shows the number of firms the accountant keeps accounts for and reminds them of good practices for tax reporting.
- **T4**. Accountant-Taxpayer (parallel) risk notification

[11] Some of the main exemptions and deductions are: tax losses depreciation, new employment additional deduction, additional deduction for expenditures related to training, technical assistance and market access, accelerated depreciation, income exemptions related to new investment, among others.

It simultaneously notifies both, accountants and taxpayers, with the same message of treatment T3. Additionally, it informs accountants that their taxpayers were notified, and vice versa, it informs taxpayers that their accountants were notified. In this treatment, at least one taxpayer per accountant is randomly notified.

- **T5**. Taxpayer penalty notification.
  It reminds taxpayers of possible penalties in case of tax evasion. It is similar to treatment T2, but instead of notifying accountants, it is taxpayers who are notified. In this treatment, at least one taxpayer per accountant is randomly notified.

The content of each notification is shown with more detail in Appendix A.[12] All notifications were sent electronically on March 10, 2016 (one month before the deadlines for corporate income tax reporting of fiscal year 2015) via the tax box system. They were sent to different groups that were selected randomly from a high-risk treatment-viable universe defined by the Ecuadorian tax administration (this universe and the corresponding groups are described in the section below). A copy of the notification was automatically sent also to their email address.

As can be seen, T1, T2, T3 notifications focus on accountants, T5 focuses on taxpayers, and T4 focus on both. The placebo notification T1 was included in order to isolate the true effect of deterrent notifications from the risk perception generated by a normal deadline reminder without any dissuasive purposes. The accountant penalty and risk notifications, T2 and T3, were a requirement of the tax administration and were structured following the traditional deterrent notifications used years over issues like accountancy, monitoring and assistance.

The last two notifications were included to evaluate the extend which notifications on accountant improve tax fulfillment more than those taxpayers. The parallel risk notification, T4, unlike T3, simultaneously notifies accountants and taxpayers with reciprocal awareness about the notification. In this sense, it allows for the evaluating whether a deterrent notification to the firm's accountant and taxpayer with a reciprocal awareness (i.e., both knows the other was notified too), is more effective than a deterrent notification to the firm's accountant. On the other hand, the taxpayer penalty notification, T5, unlike T2, contacts

[12] These notifications were designed under the criteria of tax risk experts and lawyers from the Ecuadorian tax administration.

taxpayers rather than their accountants. This notification in turn allows for the evaluating whether a deterrent notification on accountants is more effective than a deterrent notification on taxpayers.[13]

1.3.3 Data

The universe of the experiment was composed by all the firms whose accountants exclusively work for microenterprises (firms with annual revenues less than $100,000) and carry out the accounting of two or more taxpayers in the tax system (between firms and personal small businesses). This restriction was imposed by Ecuadorian tax administration for two reasons. Firstly, medium and high-income sectors are continuously intervened by semi-intensive and intensive tax enforcement policies. In this sense, there is working openness and data accessibility only for low-income firms. Secondly, accountants who work for more than one taxpayer are considered risky by tax administration,[14] so it was a requirement to delimit the universe over them.

Some statistics of this universe in the pretreatment period (fiscal year 2014) are shown in Table 1.1 for firms and accountants. In total, there were 18,465 firms and 5,945 accountants for fiscal year 2014. This universe represents nearly 20% of all corporate taxpayers in the country.

At the firm level, most are part of the service sector (52%), they are profit seeking firms (30%), and do their business in the highland region (41%).[15] At the accountant level, accountants are on average 40-years old and work for approximately three firms. Most of them are women (67%), married (54%) and possess high level of formal instruction (93%).

[13] Persuasive notifications largely target taxpayers, disregarding other parties inside or outside the firm that could have had a social, economic or legal relationship for tax fulfillment. As seen above, the accountant is one of the most important actors in this social capital due to their legal liability and accounting knowledge regarding corporate income tax reporting. So, firms' reaction to these notifications is useful for the tax administration in order to find new evidence about tax evasion behavior and design better tax enforcement policies considering taxpayers' social capital.

[14] Ecuadorian tax administration has found that the effective income tax rate decreases and the probability of tax evasion increases when multiple taxpayers are serviced by a single accountant.

[15] Ecuador is divided into four distinct regions: Amazon, Highlands, Coast, and the Galapagos Islands. Highlands are located mainly on the Andes Mountains.

Table 1.1. Baseline Summary Statistics. Fiscal Year 2014.

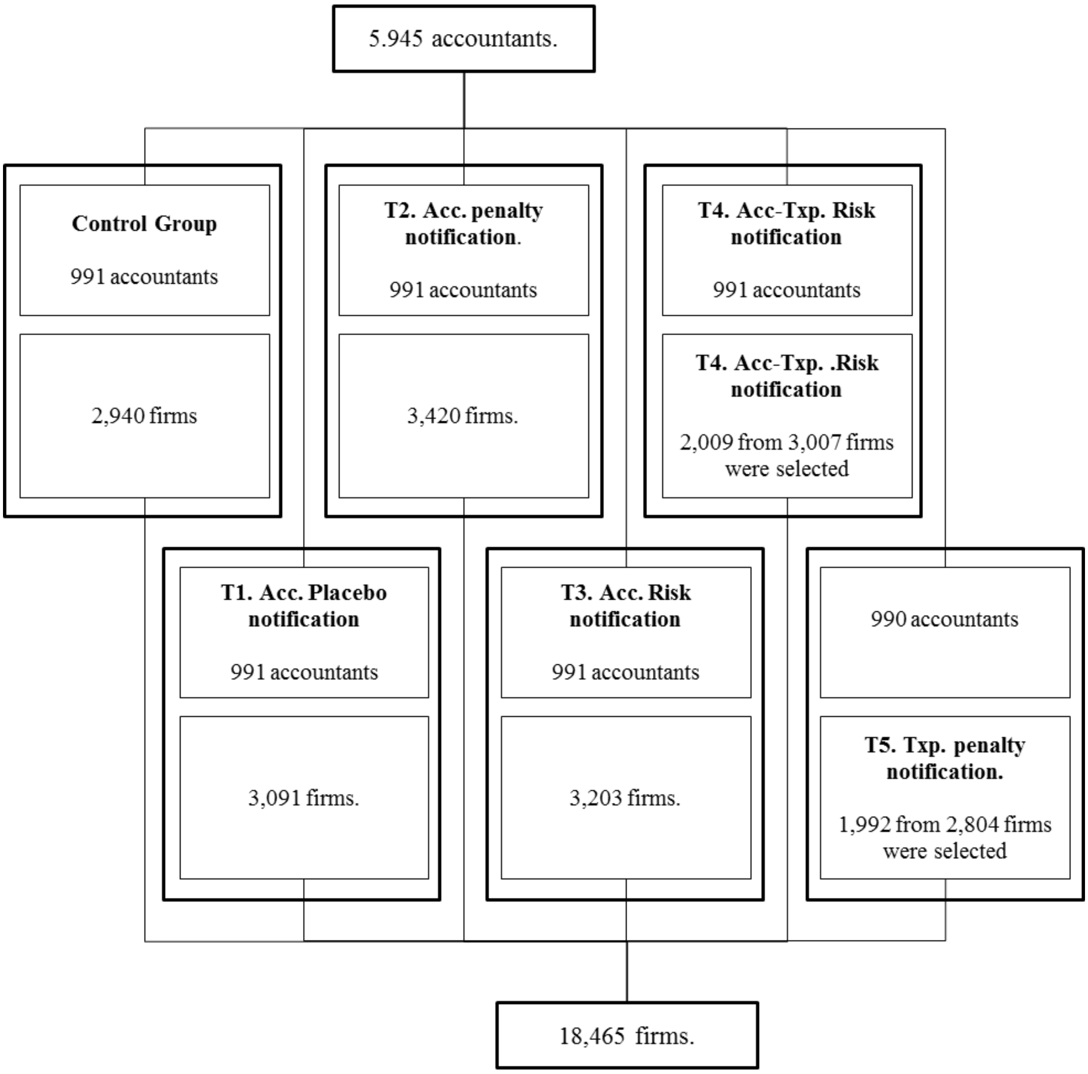
Firms level	Mean	Accountant level	Mean
Total Revenue	11.229,89	Number of firms per accountant	3,11
	(35.538,53)		(4,26)
Total Cost	13.939,48	Age	40,91
	(113.800,76)		(11,48)
Income Tax	125,13	Percentage of women	0,67
	(604,64)		(0,47)
Percentage of service firms	0,52	Percentage of married	0,54
	(0,50)		(0,50)
Percentage of profit seeking firms	0,30	Percentage of high-level instruction	0,93
	(0,46)		(0,26)
Percentage in Highlands	0,41		
	(0,49)		
Number of firms	18.465	Number of accountants	5.945

Note: This table shows the mean and the standard deviation in parenthesis for some variables in the pre-treatment period (fiscal year 2014). All monetary variables figure in USD. In the case of qualitative variables, only statistics for most relevant categories are shown.

The universe was split randomly in six equal-sized groups at the accountant level.[16] One group did not receive treatment and was used as a control group to see what would happen when firms did not receive notification (i.e., counterfactual). The other five groups were treated with the notifications from T1 to T5 (i.e., treatment groups). The following figure details the experiment design at the baseline.

[16] Due to the fact that accountants can work for several firms, a random sample was chosen at accountant level in order to have mutually exclusive treatments.

Figure 1.1. Experiment Design



Note: All notifications were sent until March 10, 2016.

Here, the control group consisted of 991 accountants that keeps the books of 2,940 firms; the treatment group T1 consisted of 991 accountants that keeps the books of 3,091 firms, and so on. It should be recalled that in treatments T1, T2, T3, all accountants were notified and none of their taxpayers was intervened. Only the last two treatments T4 and T5 exchanged the target of notification with taxpayers. Here, treatment group T4 notified simultaneously 991 accountants and 2,009 taxpayers (that were selected randomly from 3,007 firms), meanwhile treatment group T5 only notified 2,804 taxpayers (that were selected randomly from 3,007 firms) and not their accountants.

As one would expect, groups must be homogenous prior to intervention. In tables 1.2 and 1.3, almost all mean differences at both the firm and accountant levels between each treatment and control groups are statistically equal to zero for baseline variables. There are very few cases where these differences are significant despite randomization, as happened, for example, at the firm level with the percentage of firms in the highlands region in treatment T4, or at the accountant level with the number of firms for which accountant works in treatment T2.

Table 1.2. Differences between Treatments and Control Groups at Firm level.

Fiscal Year 2014.

	Difference to T1. Acc. Placebo notification	Difference to T2. Acc. Penalty notification.	Difference to T3. Acc. Risk notification	Difference to T4. Acc-Txp. Risk notification	Difference to T5. Txp. Penalty notification.
Total Revenue	-582.96	-1,032.28	-307.55	222.06	1,231.52
	(1,001.59)	(975.87)	(999.22)	(1,025.01)	(1,187.18)
Total Cost	1,195.97	-1,015.82	738.26	2,757.34*	7,115.02
	(1,897.87)	(970.98)	(1,289.56)	(1,813.05)	(4,709.41)
Income Tax	-15.56	-12.10	-1.25	13.18	22.35
	(16.10)	(16.29)	(17.28)	(17.46)	(17.94)
Percentage of service firms	-0.0175	0.0335	0.0047	-0.0055	-0.0163
	(0.0276)	(0.0263)	(0.0208)	(0.0203)	(0.0194)
Percentage of profit seeking firms	0.0401	-0.0570*	-0.0149	0.0183	0.0456*
	(0.0281)	(0.0309)	(0.0259)	(0.0251)	(0.0224)
Percentage in sierra region	0.0489	0.0035	0.0180	-0.0646**	0.0458
	(0.0362)	(0.0380)	(0.0327)	(0.0319)	(0.0308)
Number of firms	3,091	3,420	3,203	3,007	2,804

Note: Each row shows a regression of variables with dummy treatments in the pretreatment period (fiscal year 2014). Their coefficients capture the mean difference of variables between the control group and each treatment. Robust standard errors are in parenthesis. They were corrected by accountant cluster. Monetary variables figure in USD and percentage variables figures in proportions. *, **, *** denote significance at 10%, 5%, and 1% levels, respectively.

Table 1.3. Differences between Treatments and Control Groups at the Accountant level. Fiscal Year 2014.

	Difference to T1. Acc. Placebo notification	Difference to T2. Acc. Penalty notification.	Difference to T3. Acc. Risk notification	Difference to T4. Acc-Txp. Risk notification	Difference to T5. Txp. Penalty notification.
# firms	0.1524	0.4844**	0.2654	0.0676	-0.1344
	(0.1913)	(0.1913)	(0.1913)	(0.1913)	(0.1913)
Age	0.6825	0.9828*	0.8694	1.1125*	0.7054
	(0.5152)	(0.5152)	(0.5152)	(0.5152)	(0.5153)
Percentage of woman	-0.0061	0.0030	-0.0081	0.0121	-0.0114
	(0.0211)	(0.0211)	(0.0211)	(0.0211)	(0.0211)
Percentage of married	-0.0061	-0.0394*	0.0222	-0.0242	0.0036
	(0.0224)	(0.0224)	(0.0224)	(0.0224)	(0.0224)
Percentage of high level instruction	0.0111	-0.0071	-0.0071	0.0121	-0.0011
	(0.0116)	(0.0116)	(0.0116)	(0.0116)	(0.0116)
Number of accountants	991	991	991	991	990

Note: Each row shows a regression of variables on dummy treatments in the pretreatment period (fiscal year 2014). Their coefficients capture the mean difference of variables between the control group and each treatment. Standard errors are in parenthesis. Age figures are in years, while percentage variables figure in proportions. *, **, *** denote significance at 10%, 5%, and 1% levels, respectively.

The information of post-treatment period (fiscal year 2015) used for impact evaluation was collected until May 31, 2016. That is, around three months after sending notifications and around one month after the last taxpayer deadline. This date was defined in order to exclude as much as possible those firms that delay tax reporting (i.e. firms with postponing behavior) and for which the notification may have been possibly forgotten due to the long time since notification were sent.[17] At this period, there are 14,700 firms clustered in 5,131 accountants that have filed their tax return of fiscal year 2015. It means an approximate 20% reduction from the original universe in the pretreatment period.

[17] The postponing behavior weakens the impact of electronic notifications due to the temporal mitigation of the risk perception. The longer time the presentation of tax return, the wider is the gap between the notification sending date and the reporting date, and the less would be the risk perception and hence the impact on taxpayers. For example, Mcgraw et al. (1991) found that giving audio-visual information long time ago before tax reporting deadlines reduce its effect on tax returns.

For this reason, and in order to reveal significant effects, the cohort of post-treatment period was extracted until three months after sending notifications. Although this process reduces the size of the data, it does not generate attrition problems because of the circumstances explained more after (taxpayers delay their reporting not by consequence of notifications)

1.3.4 Validity factors

There are some non-negligible issues concerning internal and external validity of the experiment that should be considered before analyzing its results. When it comes to external validity, it is important to stress that the universe of the experiment is a non-random segment defined by the Ecuadorian tax administration (firms whose accountants exclusively work for microenterprises and keep the books of two or more taxpayers). Therefore, the empirical results shown further down cannot be extrapolated or generalized to the whole population of taxpayers. They must be used strictly to explain the behavior of the specified taxpayer segment. Nonetheless, it could be said that the results show an extreme case regarding firms' responses in the small business sector because the selected firms exhibit a high-risk level for income tax fulfilment according to tax expert criteria.

In terms of internal validity, the randomness of treatments and the incorporation of a placebo intervention in the field design, allow for the estimation of a true causal effect of deterrent notifications on income tax reporting. As shown above, the treatments and control groups are statistically balanced as a result of the random selection. With no significant ex-ante difference between them, any ex-post difference can be attributed to the impact of notifications. So, the identification of causal effects in a firm's tax reporting is possible by simply comparing groups in the post-treatment period. Moreover, the addition of a placebo allows for the observation of whether these effects are significantly produced by a higher risk perception on deterrent messages or by non-dissuasive notifications that could make taxpayers think they are being monitored.

It should also be noted that the country's tax administration did not intervene the universe of the experiment by any enforcement policy after notifications were sent (on March 10, 2016) and before tax information was collected (May 31, 2016). This ensures that the estimated effects are exclusively attributed to the notifications. On the other hand, although external factors such as the deceleration of Ecuador's economy and the earthquake that took place in 2016 could have influenced companies' activities and therefore their tax returns, the randomness of the experiment guarantees that both the control group and the treatment groups were affected equally by these factors, thus generating no bias on estimations.
Despite these advantages, there is one important issue that potentially threatens the internal validity of results: the experimental mortality or attrition due to postponing behavior of firms.

As mentioned before, about 20% of the original universe has not filed the tax return prior May 31, 2016, when data were downloaded for the post-treatment period.

Since notifications did not require accountants and taxpayers to provide additional information on their bookkeeping practices, neither required the fulfillment of any new tax obligation, it is reasonable to expect that taxpayers who did not report taxes before the deadline were motivated not to do so by external factors (rather than by the experiment itself).

In order to test this fact, a Probit model is estimated for the probability of reporting before May 31, 2016. As seen in table 1.4, the placebo treatment is the only one that significantly influences a firm's postponing behavior while the other treatments have no impact. It means the proportion of firms that filed taxes until the post-treatment date were statistically equal between treatments and control groups, except for the placebo group.

Table 1.4. Probit of Reporting before May 31, 2016. Fiscal Year 2015.

T1. Acc. Placebo notification	0.0338**
	(0.0149)
T2. Acc. Penalty notification	0.0060
	(0.0206)
T3. Acc. Risk notification	0.0155
	(0.0153)
T4. Acc-Txp. Risk notification	0.0002
	(0.0156)
T5. Txp. Penalty notification	0.0217
	(0.0141)
Number of observations	18,465

Note: This table shows the marginal effects of reporting in post-treatment period (fiscal year 2015). All treatment variables are dummies. Robust standard errors are in parenthesis. They were corrected by accountant cluster. *, **, *** denote significance at 10%, 5%, and 1% levels, respectively.

It is important to recall the placebo notification was the unique treatment that just provided information on tax reporting deadlines, so one would expect it influences on postponing behavior more than others. The remaining notifications also had the same reminder at the beginning, however their content was more focused on dissuasive messages (years of imprisonment, number of taxpayers to whom accountants work with, common knowledge, etc.). Therefore, one would expect that these notifications persuade taxpayers/accountants to report taxes honestly rather than doing them on time.

1.3.5 Estimation strategy

The impact estimation of each treatment on declared taxes, revenues and costs was done through a simple regression model. Here, I used the midpoint relative change of tax reporting variables as outcome variable.[18] This indicator has the advantage of enclosing the variable's percentage rate within a bounded interval from -200% to 200%, thus diminishing the variance of estimators. Also, it can be calculated for variables with initial null values, and, among other properties, it is not sensitive to scale, symmetric and robust to outliers.

Under these considerations, the following lineal model was estimated:

$$\Delta^{\%} Y_{ij} = \alpha + \beta T_j + \theta X_i + e_{ij}$$

where $\Delta^{\%} Y_{ij}$ is the midpoint relative change of tax reporting variables declared by the firm i with the accountant j, between fiscal years 2014 and 2015; T_i represents the vector of treatment variables according to treatments described above; X_i is the vector of covariables and e_{ij} is the error. α, β, θ are the coefficients of the model.

The treatment variables from T1 to T3 are dummies and indicate whether accountant j was notified. In contrast, the treatment variables T4 and T5 are continuous and represent the proportion of firms that were notified for each accountant j. Since at least one firm is notified randomly in both treatments, these proportions are strictly positive and they approach to 1 as

[18] The midpoint relative change measures the absolute difference between two values in terms on the mean of both. It is defined by:

$$\Delta^{\%} Y = h(y_1, y_0) = \frac{(y_1 - y_0)}{\bar{y}}$$

It is proved that (Tornqvist, Vartia, and Vartia 1985; Lorenzen 1990):

1. $h(y_1, y_0) = 0$ if $y_1 = y_0$
2. $h(y_1, y_0) > 0$ if $y_1 > y_0$
3. $h(y_1, y_0) < 0$ if $y_1 < y_0$
4. $h(y_1, y_0) = -h(y_0, y_1)$
5. $|h(y_1, y_0)| \leq 2 \; \forall y_1, y_0$

more firms are notified. This feature not only makes treatments T4 and T5 comparable to the others, but also has the advantage of introducing intensity to these interventions.[19] It should be noted the variables T1 to T5 are independent and exogenous due to the randomness of the experiment.

The covariables X_i were divided in three sets. firms' characteristics (economy activity, region and company type), accountant's characteristics (age, gender, civil status and instruction) and time variables (date until when tax return must be filed). All these variables were extracted from tax administration data base and were gradually incorporated in the regression model in order to check the robustness of the effect estimation.

In the statistical model described above, the effect of treatments is identified through the set of coefficients β. These coefficients quantify how high the relative change of tax reporting variables in treatments groups compare to the control group. Therefore, their estimation is the mainly interest of this paper.

Due to the experimental design used, it should be noted that observations in the model are independent between firms with different accountant and not among firms that have accountants in common. As such, estimations were corrected by using cluster-robust errors. This correction increases the probability of finding nonsignificant effects due to cluster characteristics (for example, a low cluster size and/or a high intra-cluster correlation), however it is consistent with accountant interventions made in the experiment.

1.4 Results

1.4.1 Overall effects

The effects of each treatment on declared total revenue, total cost and income tax are shown in tables 1.5, 1.6 and 1.7, respectively. In addition, the differences in effects between any pair of treatments are shown in the Appendix B.

[19] For example, if an accountant keeps the books of five firms and only three of them was notified (either by T4 or T5), then the accountant practically gets the three fifths of the treatment that he would receive if all their firms were notified.

The more remarkable effects appeared on total revenue. Here, the accountant penalty (treatment T2), accountant risk (treatment T3) and parallel risk (treatment T4) notifications had a positive and statistically significant impact. The stronger effect was provided by parallel risk notification (treatment T4) which increased revenue at the 10% at 5% significance level. The accountant penalty notification (treatment T2) increased revenue by 8.4%, at 1% significance level (most significant). Finally, the accountant risk notification (treatment T3) increased revenue by approximately 5.2 % with a lower significance level, 10%. The rest of the treatments had a positive effect but were statistically equal to zero (Table 1.5).

Table 1.5. Impact Estimation on Declared Total Revenue. Fiscal Year 2015

	Spec. 1	Spec. 2	Spec. 3	Spec. 4
T1. Acc. Placebo notification	0.034	0.027	0.023	0.024
	(0.029)	(0.029)	(0.030)	(0.030)
T2. Acc. Penalty notification	0.081***	0.084***	0.082***	0.083***
	(0.029)	(0.028)	(0.029)	(0.028)
T3. Acc. Risk notification	0.052*	0.052*	0.047	0.049
	(0.031)	(0.030)	(0.030)	(0.030)
T4. Acc-Txp. Risk notification	0.101**	0.095**	0.098**	0.101**
	(0.043)	(0.042)	(0.043)	(0.043)
T5. Txp. Penalty notification	0.043	0.032	0.031	0.031
	(0.039)	(0.039)	(0.040)	(0.039)
Firm's covariables	no	yes	yes	yes
Accountant's covariables	no	no	yes	yes
Time covariables	no	no	no	yes
Number of observations	14,700	14,700	14,700	14,700

Note: Each column shows a regression of total revenue relative change on treatments in the post-treatment period (fiscal year 2015). The specifications spec. 1 to spec. 4 include gradually the covariable's sets (firm's variables, accountant's variables and time variables). Treatments variables from T1 to T3 are dummy variable, while T4 and T5 are continuous variables. Robust standard errors are in parenthesis. They were corrected by accountant cluster. *, **, *** denote significance at 10%, 5%, and 1% levels, respectively.

Interestingly, the fact that both accountants and taxpayers are notified by tax administration (treatment T4) increased more revenues than notifying exclusively accountants with a similar message (treatment T3), however this difference is not significant (see Appendix B.1). This shows a possible interaction between both parties when they are simultaneously contacted by tax administration in order to reduce the risk perception about notifications. This interaction is intensified not only by reciprocal awareness of the treatment (each party knows that the other was notified), but also by accountants' private information shown in notifications (number of firms for which they work for). The fact that both know that the tax administration has private information (in this case about accountant business) can increase taxpayers' risk perception because it could make them aware that the tax administration is strong and capable of obtaining any type of economic information for fiscal purposes.

It is also worth noting the penalty notification had a greater effect on accountants (treatment T2) than on taxpayers (treatment T5), however, as before, this difference is not significant (see Appendix B.1). This result makes sense because accountants are more risk-averse since they could work for several firms making their tax reporting. Hence, their legal liability is greater than a single taxpayer could have. This premise is analyzed in more detail latter on with interaction terms.[20]

When it comes to cost reporting, only accountant penalty notifications (treatment T2) had a significant impact. It increased the total cost by approximately 6.5% at the 5% significance level. The rest of the treatments had a positive effect but were statistically equal to zero (Table 1.6).

---

[20] These results are consistent with the agency model made by Crocker and Slemrod (2005). As noted in literature review, the main factor that stress the effect of penalties on accountants is the information asymmetry regarding the accounting rules to reduce the tax payment. The privilege that accountants have when accessing this kind of information, and their legal liability on firms' tax reporting (i.e. the possibility of being penalized), makes them more sensitive than taxpayers to any risk signal, such as, for example, a persuasive notification. In this sense, it is expected that firms' income reporting improves more when accountants are notified rather than taxpayers themselves, or even more when both accountants and taxpayers are notified simultaneously with reciprocal knowledge.

Table 1.6. Impact Estimation on Declared Total Cost. Fiscal Year 2015

	Spec. 1	Spec. 2	Spec. 3	Spec. 4
T1. Acc. Placebo notification	0.022	0.018	0.013	0.014
	(0.030)	(0.030)	(0.030)	(0.030)
T2. Acc. Penalty notification	0.070**	0.069**	0.065*	0.066**
	(0.030)	(0.029)	(0.029)	(0.029)
T3. Acc. Risk notification	0.044	0.043	0.036	0.039
	(0.032)	(0.032)	(0.032)	(0.032)
T4. Acc-Txp. Risk notification	0.035	0.031	0.034	0.038
	(0.043)	(0.042)	(0.042)	(0.042)
T5. Txp. Penalty notification	0.030	0.020	0.018	0.019
	(0.040)	(0.040)	(0.040)	(0.040)
Firm's covariables	no	yes	yes	yes
Accountant's covariables	no	no	yes	yes
Time covariables	no	no	no	yes
Number of observations	14,700	14,700	14,700	14,700

Note: Each column shows a regression of total cost relative change on treatments in the post-treatment period (fiscal year 2015). The specifications spec. 1 to spec. 4 include gradually the covariable's sets (firm's variables, accountant's variables and time variables). Treatments variables from T1 to T3 are dummy variable, while T4 and T5 are continuous variables. Robust standard errors are in parenthesis. They were corrected by accountant cluster. *, **, *** denote significance at 10%, 5%, and 1% levels, respectively.

Despite the possible theoretical contradiction, these results are not surprising because firms use cost mechanisms to reduce tax payments. If they increase the declared revenue as a response to tax notification, they could increase the declared cost in order to eliminate any tax increases.[21] In the context of the present study, it is likely that an accountant uses this mechanism in lower-risk situations, when deterrent notifications are standard and have been known for a long time by taxpayers without any subsequent tax audit. This seems to happen with the accountant penalty notification (treatment T2), which had a similar format to other tax notifications implemented previously by the Ecuadorian tax administration. For the other

[21] This behavior is similar to tax substitution found by Carrillo, Pomeranz, and Singhal (2017), but without third-party information.

treatments, cost overreporting is reduced due to their novelty, either through notifications that included some private information about accountants (treatment T3), through notifications that simultaneously contacted two parties of firms' social capital (treatment T4), or through notifications that contacted taxpayers with same accountants (treatment T5).

All the patterns above explain the effects on declared income tax. As can be seen in Table 1.7, the parallel risk notification (treatment T4) is the unique treatment that had a significant positive impact as it increased the declared income tax by approximately 8.3% at the 5% significance level. But not only that, this effect was significantly greater than that obtained by notifying only accountants without knowledge of their taxpayers (treatment T3), as seen in Appendix B.3.

Table 1.7. Impact Estimation on Declared Income Tax. Fiscal Year 2015

	Spec. 1	Spec. 2	Spec. 3	Spec. 4
T1. Acc. Placebo notification	-0.022	-0.025	-0.028	-0.026
	(0.033)	(0.033)	(0.033)	(0.033)
T2. Acc. Penalty notification	0.018	0.019	0.019	0.021
	(0.025)	(0.025)	(0.025)	(0.025)
T3. Acc. Risk notification	0.010	0.009	0.007	0.008
	(0.027)	(0.027)	(0.027)	(0.027)
T4. Acc-Txp. Risk notification	0.084**	0.084**	0.084**	0.084**
	(0.035)	(0.035)	(0.035)	(0.035)
T5. Txp. Penalty notification	0.051	0.047	0.047	0.047
	(0.036)	(0.036)	(0.036)	(0.036)
Firm's covariables	no	yes	yes	yes
Accountant's covariables	no	no	yes	yes
Time covariables	no	no	no	yes
Number of observations	14,700	14,700	14,700	14,700

Note: Each column shows a regression of income tax relative change on treatments in the post-treatment period (fiscal year 2015) The specifications spec. 1 to spec. 4 include gradually the covariable's sets (firm's variables, accountant's variables and time variables). Treatments variables from T1 to T3 are dummy variable, while T4 and T5 are continuous variables. Robust standard errors are in parenthesis. They were corrected by accountant cluster. *, **, *** denote significance at 10%, 5%, and 1% levels, respectively.

As noted before, the interaction between accountants and taxpayers, the reciprocal awareness of both parties, and the accountant's private information are the main factors in this notification which reduced revenue underreporting and cost overreporting. As a result, a better tax return was generated.

It should be noted that the placebo treatment had no significance in all estimations shown in table 1.6, 1.7 and 1.8. That means the results described above are directly related to the deterrent messages, not by the fact that taxpayers might think they are being monitored by tax administration when they are not. This evidence strengthens the reliability of estimations.

In summary, research findings present some interesting conclusions relating to firms' tax behavior. On the one hand, between all notifications in the experiment, only parallel-risk notifications (treatment T4) had a statistically significant effect on income tax. As a matter of fact, these notifications produced a greater effect than risk notifications sent exclusively to accountants (treatment T3). This suggests that accountants and taxpayers interact on tax reporting and reach an agreement in order to reduce risk perception effectively.

On the other hand, penalty notifications on accountants (treatment T2) had the most significant effect on declared revenue. These notifications were even more effective at increasing declared revenue than notifications on taxpayers only (treatment T5), but with a non-significant difference. Despite that, these notifications did not produce a significant impact on income tax due to an overreporting cost mechanism that cancelled out the previous effect. This was likely due to electronic sending and the notification's standard format used by tax administration that lessen the risk perception of taxpayers.

1.4.2 Interaction effects

In this section, only the heterogeneity of parallel risk notification's impact (treatment T4) is analyzed. To do so, the treatment is interacted with characteristics of both, taxpayers and accountants.

Table 1.8 shows how the effect of parallel-risk notification (treatment T4) varies in relation to firm size, according to revenues and cost declared in fiscal year 2014. Here, results show that the higher the revenues and costs, the greater the effect on income tax, with a significant interaction terms at the 1% significance level. Due to the fact that the main effect of the treatment is nonsignificant, it could be stated that these results are mainly driven by the size of the firm.

This evidence is consistent with the microeconomic theory of tax evasion, which supports that tax evasion is greater (as well as risk perception) when taxpayers' incomes are higher. This means that any dissuasive policy would likely produce more effective tax changes for firms whose revenues and/or costs are higher.

Table 1.8. Interaction of Parallel Risk Notification (Treatment T4)
by Firm's Variables

	Spec. 1	Spec. 2
Interation terms		
T4 x Log(Txp. Revenue)	0,022***	
	(0,0068)	
T4 x Log(Txp. Costs)		0,023***
		(0,0063)
Main Effects		
T4. Acc-Txp. Risk notification	0,006	-0,007
	(0,0326)	(0,0337)
Log(Txp. Revenue)	-0,035***	
	(0,0023)	
Log(Txp. Costs)		-0,025***
		(0,0022)
Number of observations	14.700	14.700

Note: Each column shows a regression of income tax relative change on treatments in the post-treatment period (fiscal year 2015). Spec. 1 includes the interaction term for firm's revenues and Spec. 2 includes the interaction term for firm's costs. All covariable's sets were included for estimation (firm's variables, accountant's variables and time variables). Table only shows estimations for treatment T4. Acc-Txp risk notification. Robust standard errors are in parenthesis. They were corrected by accountant cluster. *, **, *** denote significance at 10%, 5%, and 1% levels, respectively.

Table 1.9 displays impact estimations when the number of firms for which accountants work for or their age varies. As shown, the more taxpayers accountants keep books or the younger they are, the greater the effect will be on income tax. Despite the expected signs, only the interaction term related with the number of firms is significant at the 10% level. In addition, as it occurred before, the main effect of the treatment is nonsignificant.

Table 1.9. Interaction of Parallel Risk Notification (Treatment T4) by Accountant's Variables

	Spec. 1	Spec. 2
Interation terms		
T4 x Log(Acc. #firms)	0,064*	
	(0,0357)	
T4 x Acc. Age		-0,031
		(0,1146)
Main Effects		
T4. Acc-Txp. Risk notification	-0,021	0,201
	(0,0690)	(0,4290)
Log(Acc. #firms)	-0,007	
	(0,0109)	
Acc. Age		0,096
		(0,2071)
Number of observations	14.700	14.700

Note: Each column shows a regression of income tax relative change on treatments in the post-treatment period (fiscal year 2015). Spec. 1 includes the interaction term for firm's revenues and Spec. 2 includes the interaction term for firm's costs. All covariable's sets were included for estimation (firm's variables, accountant's variables and time variables). Table only shows estimations for treatment T4. Acc-Txp risk notification. Robust standard errors are in parenthesis. They were corrected by accountant cluster. *, **, *** denote significance at 10%, 5%, and 1% levels, respectively.

These results make sense. It is expected that the effect of notifications on accountants is high when the number of taxpayers for whom they work for is important, as their responsibility in complying with tax obligations and their perception of risk increases. By contrast, the inverse effect regarding to accountant's age may evidence their experience with carrying out legal or illegal practices for reducing tax liability, which would help to mitigate a company's risk perception.

1.5 Conclusions

Tax evasion could be analyzed as a social economical phenomenon through which several parties interact in order to reduce tax payment. Here, accounts are key agents due to their privileged information on accounting rules and how to apply them, their liability in the fulfillment of tax obligations, the fact that they can work for several firms, and the possible compensation mechanisms to mitigate the risk of being detected.

Accountants has been empirically invisible party in the design of tax enforcement policies. Although there have been an increasing number of impact evaluations of deterrent policies on firm's tax behavior, the effect of deterrence actions on accountants and their participation in tax evasion remains unknown beyond the theoretical implications of agency models.

This study has the first empirical evidence on the causal effect of deterrent notifications on accountants. Through an experiment conducted in Ecuador's tax system, it was shown that simultaneous deterrent notifications on both accountants and taxpayers with reciprocal awareness (treatment T4) increase firms' declared income tax by approximately 8.3% at the 5% significance level. This effect was the only significant one in the experiment, it was even significantly higher than effect caused by the notifications focused on accountants only (treatment T3). Moreover, it was shown that penalty notifications on accountants (treatment T2) increased firms' declared revenue by 8.4% at the 1% significance level. This effect was even higher than effect caused by the penalty notifications focused on taxpayers only (treatment T5), but with a non-significant difference. It should be noted that despite the fact that penalty notifications on accountants was more effective in firms' declared revenue, they did not generate a significant impact on declared income tax.

These results suggest that accountants have an active role in a firm's tax reporting. One the one hand, there is evidence of interaction between the accountant and taxpayer when both are notified, which increases the perceived risk by firms and stimulates a better tax reporting. On the other hand, there is a cost mechanism through which accountants apparently cancel the effect of notifications. Even though firms increase the declared revenue when their accountants are notified, they seem to overreport costs in order to have less tax payment.

The systemic relationship between accountants and taxpayers, and the evasion mechanisms by cost overreporting are innovative clues to understand accountants' behavior in a firm's tax reporting and to design better tax enforcement policies. It is recommended to extend this kind of analysis to other parties that are involved in the social capital of the firm, such as suppliers, owner's familiars or shareholders, and evaluate their reaction to tax monitoring.

Chapter 2

Productivity Shocks Diffusion in Firms Networks with Imperfect Competition and Bankruptcy*

2.1 Introduction

Many theoretical and empirical studies use the concept of production network or network of firms to analyze the propagation of microconomic shocks in an economy and their effect on macroeconomic aggregates (Carvalho 2010; Acemoglu et al. 2012; Stella 2015; Acemoglu et al. 2016; Carvalho et al. 2016; Acemoglu, Ozdaglar, and Tahbaz-Salehi 2017).

The contributions on this field assume markets with perfect competition where identical firms act as price takers in walrasian equilibrium setting. Among other things, this assumption implies that the main mechanism of shock transmission (for example, a negative shock on the productivity of a group of firms) is changes on the price of the inputs these firms produce, and their subsequent substitution by other inputs (usually a partial substitution). In spite of the unquestionable importance of this mechanism in the diffusion of the shock, its effect tends to diminish or lessen as the “shock wave” expands on the production network, since inputs (affected by the shock) are only a part of each firms’ costs.

Assuming the shock is negative, there is another important effect on the economy, namely the possibility of firm exit. In a Walrasian equilibrium setting, perfect competition between input producing firms implies that if one firm exits the market, its production share will be taken up by the remaining firms and consequently, there would be a shift in the demand for inputs and labor. Moreover, if the production function of these firms is homogenous of degree one, their exit from the market won’t even give rise to a change in prices. However, in the presence of a negative shock, a recurring

---

* This research was co-authored by Wilson Pérez, Phd. I thank Victor Aguiar, Carlos Uribe and Felipe Brugués for helpful comments and discussions.

This research was carried out using the research computing facilities and/or advisory services offered by Scientific Computing Laboratory of the Research Center on Mathematical Modelling: MODEMAT, Escuela Politecnica Nacional - Quito.

concern for policy makers and in general for economic agents is the possibility of firm bankruptcy and a subsequent decrease in employment and production. Is this a valid concern? According to the Walrasian model, it is not.

The present work studies the propagation of a productivity shock in production networks characterized by imperfect competition with the possibility of firm exit. To do so, an economic model based on a network is built, where nodes represent firms and edges represent input supply from one firm to another. It is a weighted, directed, and random (as shall be explained later) network. The modelled economy also includes a representative household which has a two-fold relationship with the firms: as labor provider and purchaser of final goods. The economy is assumed to be closed.

The firms are characterized by: (1) a production function which, in principle, may represent any production set that fulfills the properties of free disposal, no free lunch, a closed production set and the possibility of inaction. [22] (2) An evaluation function that represents the firm's weighing between the variation in its expected utility (derived from hiring one input producing firm over another) and the variation in the risk this decision entails. The input-buying firm's evaluation of this trade-off between a higher expected utility and a higher risk is a consideration we add to the problem of expected utility maximization; the perceived risk of contract failure related to each potential input supplier and the cost involved for the buying firm. This way, due to previous contact between suppliers and the buying firm, for regular suppliers both variables are perceived as lower compared to new suppliers; consequently, in order to be competitive, the latter must reduce their prices.

As will be seen in section 3.1, due to characteristics (2) and (3), the firm assigns a probability (which can be zero or one) to the decision of hiring a specific supplier; in other words, in this approach the decision is random. This makes it possible to model different degrees of competitiveness in the specific market a buying firm faces when searching for a specific input. Hence, a buying firm's perception of a very high risk when hiring a supplier different from its usual supplier would lead to the usual supplier

[22] However, the demonstration of the existence of equilibrium in the model here presented is carried out for a Leontief technology in every one of the inputs, including labor. See Appendix E.

to behave like a monopolist when facing this buying firm. This could happen when the buying firm requires an input with very specific characteristics, when product quality is very hard to know *ex ante,* and/or when a mistake in the details, time or place of delivery are very possible. On the other hand, if the input has standard characteristics which can be easily verified *ex ante* and the costs of failures in the input provision are low, we would be dealing with a competitive market for this particular input, which is why the supplier would behave as a price accepting firm.

Built this way, the economy is a probabilistic space where a unitary event represents a subgraph in a production network in which nodes represent firms that have not gone bankrupt (see below for the probability of firm exit or bankruptcy) and the edges are the input provision contracts between one firm and another (this includes prices and quantities) which have been effectively assigned. The probability of this single event depends on characteristics (1), (2) and (3) of the nodes (see above) and on the prices that each input supplier sets for its output.

In this model, the sequence of events is as follows: the firms that produce intermediate goods (inputs) set the selling prices that maximize their expected benefits taking into account that these prices impact the probability of being assigned (or not) the input provision contract and considering the demand function of their potential buyers. We assume that each firm produces only one intermediate good. When making this decision, each firm assumes all other firms’ prices as given, so the first equilibrium in the economy is a Nash equilibrium. In this equilibrium, each firm sets the prices that maximize its expected benefits, given the prices set by the rest of the firms. Additionally, the market for final goods is a perfect competition market and therefore, firms are price takers. Lastly, the representative household maximizes its expected utility by means of its demand for final goods and labor supply.

Given technologies and potential suppliers (with their risk related characteristics) for each buying firm, prices fixed previously through the Nash equilibrium determine the probability in the probabilistic space of the production network. In this space, for each event realization, wages are endogenous, and each firm decides the quantity of labor and inputs it will hire and purchase, such that total market supply equals the market demand in each market. This is what we refer to as General Market Equilibrium (GME), that is, the set of prices that constitute a Nash equilibrium *ex ante*, and of quantities and wages that guarantee the *ex post* equilibrium between supply and demand.

In this model, firm exit or bankruptcy is also a random event. Hence, the probability of a firm going bankrupt is greater the higher the number of periods in which the firm obtained negative results. This fact is incorporated into the model through a stochastic process in which each time period is determined by the GME of the firms that have survived until that moment.

In this model, a negative shock in a firm's productivity could have the following effects. First, the affected firm would have to increase its prices, since producing its output becomes more costly. This allows its competitors a margin to increase their prices, despite not having suffered from the shock's impact. Second, confronted with its price increases, the probabilities of the affected firm being hired by its potential buyers would decrease, which in turn would affect the firms linked to it. Therefore, its downstream buyers would have to face higher prices for this input, and subsequently, withstand an increase in their prices and a reduction in the probabilities of being hired. Third, having lost a buyer, the affected firm's upstream regular suppliers may expect a reduction in their sales and must reduce their prices in search for higher chances of gaining other contracts; both factors reduce their profits. If these negative effects are large enough to give rise to more firm exits in the production network, the process repeats itself, being one of the mechanisms of propagation of the initial shock. To evaluate these effects, numeric simulations of the theoretical model over a simple network production are carried out.

The present work relates to various theoretical studies on the propagation of microeconomic shocks in production networks. In particular, this study contributes to the discussion that is developed in the work of Acemoglu et al. (2012), on cascade effects and microeconomic fluctuations in competitive markets. Acemoglu analyzes the propagation of microeconomic shocks through a general equilibrium model with a representative household and various economic sectors. Here, sectors are price takers and use Cobb-Douglas technologies with constant returns to scale. These technologies generate zero profits and linear input costs for each sector. Its main results show that a productivity shock that affects a specific sector may propagate downstream to the rest of sectors in the economy (i.e. from buyer to buyer), depending on each sector's degree of participation as input providers in the network. Many research papers on the diffusion shocks in economic networks adopt this theoretical approach (Carvalho and Gabaix 2013; Stella 2015; Acemoglu, Ozdaglar, and Tahbaz-Salehi 2016; Carvalho et al. 2016; Acemoglu, Ozdaglar, and Tahbaz-Salehi 2017; Atalay 2017).

The approach used in the present work is more closely related to the work of Baqaee (2016) on cascade effects in economies with market failures. Baqaee analyzes the propagation of microeconomic shocks on production networks with non-competitive markets, and firm entry and exit. To do so, they develop a model of price setting in which market price is dependent on the profit margin and the number of firms in each sector. In this model, there exists no price discrimination between demanding sectors and the assumption of a representative firm with constant returns to scale is upheld. The model's results show that a productivity shock may generate simultaneous upstream and downstream cascade effects, multiplying the impact on the network beyond cases with perfect competition. This work is one of the first ones in discussing market failures on the propagation of shocks in production networks; nevertheless, some assumptions from the perfect competition framework remain (for example, zero profits and no price discrimination) which relax the production network's behavior and the shock transmission. Similarly, eventhough this work considers product differentiation as the main reason for firm exit from the industry (and therefore, for firm bankruptcy), its inclusion in the model is dimmed by the homogeneity of the economic sectors.

Our research also contributes to the construction of new methods for modeling choice under risk. In the conventional risk aversion model, the agent seeks to maximize the value of its expected utility. This way, the agent's preferences regarding the risk involved in each presented option are represented through the curvature in the utility function (i.e. the absolute degree of risk aversion). However, the solution to this problem is a corner solution, meaning it assigns 100% probability to the option that generates the largest expected utility. In this sense, it is a deterministic solution to a random problem. Additionally, as stated by Donoghue and Somerville (2018), the conventional risk aversion model discards preferences which are seemingly plausible due to a calibration problem when dealing with options that have different returns (e.g., the model does not allow the agents to exhibit a high risk aversion when there are options with low and moderate returns at the same time). Notice that even though a project may be much riskier that another project, the former may be chosen because its expected utility is slightly higher than that of any other alternative. Our work puts forward a design where the agent evaluates each project by how much additional expected utility can be derived from choosing it, versus the project's implicit risk. In this sense, the potential increases in the expected utility are weighted against a worst-case scenario situation.

The rest of the document is organized in the following way. The second section briefly reviews the theoretical contributions in the analysis of the diffusion of microeconomic shocks. The third section summarizes the fundamentals of the problem of probabilistic choice, the behavior of each one of the agents in the production network, the formulation of the General Market Equilibrium (GME) approach and firm bankruptcy in the model. The fourth section carries out a simulation exercise based on a production network that is not real and analyzes how productivity shocks are propagated in the productive system. Finally, the fifth section presents the study's conclusions.

2.2 Literature review. The theory of the diffusion of microeconomic shocks in production networks

The application of networks in economics is wide. There exist various works in areas like experimental economics, the formation of strategic networks, information and learning diffusion, the labor market, social interrelations and development, negotiation and market power, international commerce and international networks, systemic risk and finance, among others (Jackson 2014).[23] At present, a prominent field of research is the analysis of microeconomic shock diffusion in a production network. Here, one of the main objectives has been to analyze the validity of Lucas' argument on the formation of economic cycles. This argument, also known as the law of large numbers or the argument of diversification, states that microeconomic shocks generate non-significant effects on the variations in macroeconomic aggregates, since these shocks "average out" and become negligible when the number of firms in an economy is large

[23] Some applications in these areas are explained in detail in Jackson (2008, 2009), Easley y Kleinberg (2010), Bramoullé, Galeotti, and Rogers (2016) Goyal (2016).

(Lucas 1977). Consequently, the likelihood of the emergence of significant fluctuations on macroeconomic aggregates as a result of microeconomic shocks is small.

In this field, diverse works on production networks stand out. From a theoretical standpoint, the following studies can be found: Acemoglu, Ozdaglar, and Tahbaz-Salehi. (2010), Carvalho (2010), Acemoglu et al. (2012), Jones (2013), Carvalho et al. (2016), Bigio and La'O (2016), Grassi (2017), Acemoglu, Ozdaglar, and Tahbaz-Salehi (2017), D. R. Baqaee (2018) y D. R. Baqaee and Farhi (2019).[24] The main objective of this work is to determine the conditions under which a microeconomic shock on the production network can generate downstream and upstream[25] effects, by using a general equilibrium model. These studies will be described in what remains of this section.

Acemoglu, Ozdaglar, and Tahbaz-Salehi. (2010) analyze the relationship that exists between the production network's structure and the volatility of macroeconomic aggregates, using a static version of the equilibrium model proposed by Long and Plosser (1983).[26] This model considers a representative household and various economic sectors. On the one hand, household preferences are assumed to be Cobb-Douglas. On the other hand, economic sectors use Cobb-Douglas production functions with constant returns to scale. Under these assumptions the authors demonstrate that idiosyncratic shocks at the sectoral level can generate downstream cascade effects. In other words, these shocks can be transmitted to the buyers of affected sectors, in turn to their buyers, and so on. This phenomenon takes place in networks for which the degree of nodes' distribution fulfills the power law.[27] Specifically, these networks present two important characteristics for the diffusion of shocks. First, there are sectors with a large

[24] From an empirical standpoint, there also exist various works that show the propagation of microeconomic shocks in the production network of various countries and their effect on macroeconomic aggregates. Among the most recent studies include Foerster, Sarte, and Watson (2011), Carvalho and Gabaix (2013), Aobdia, Caskey, and Ozel (2014) Di Giovanni, Levchenko, and Mejean (2014), Carvalho (2014), Acemoglu, Akcigit, and Kerr (2015), Bartelme and Gorodnichenko (2015), Stella (2015), Acemoglu et al. (2016), Barrot and Sauvagnat (2016), Carvalho et al. (2016), Atalay (2017), y Boehm, Flaaen, and Pandalai-Nayar (2019).

[25] A downstream effect refers to a shock diffused from a firm or sector to its buyers, their buyers' buyers and so on. On the contrary, an upstream effect refers to a shock spread from a firm or sector to its suppliers, their suppliers' suppliers and so on.

[26] The Long and Plosser model (1983) is the methodological basis for most analysis of the diffusion of microeconomic shocks in a productive network.

[27] This distribution is characterized for being strongly biased to the right, in other words, there exists a small number of nodes in the network that have a high number of close neighbors. Several empirical studies show that various economic networks present this structural regularity. See Jackson (2008)

number of buyers in the economy (first order interrelations); and second, there are highly central sectors in the production network which have common suppliers (superior degree interrelations). According to the authors, the latter is essential in explaining the aggregate volatility in the economy, since any negative shock that affects the suppliers will decrease their buyers' demand, which will multiply the effect onto the rest of sectors, due to their high centrality in the network.

Carvalho (2010) shows how idiosyncratic shocks in the economic sectors can generate fluctuations in the macroeconomic aggregates. To do so, he uses a general equilibrium model with a representative household and various sectors with Cobb-Douglas technologies and preferences. It is assumed that households have a preference for leisure, which is why labor supply is endogenous. Based on this model, the author finds that sectoral shocks may propagate in the economic system and impact macroeconomic aggregates. This outcome occurs in production networks where demand is not very diversified and a reduced group of sectors concentrate supply. It is shocks in sectors with these characteristics which are multiplied rapidly in the production network and which widen fluctuations in the macroeconomic aggregates. This result is similar to the one arrived at by Acemoglu, Ozdaglar, and Tahbaz-Salehi (2010) but for networks structured differently (for example, star-shaped networks and complete graphs).

Acemoglu et al. (2012) is possibly one of the main references for the analysis of microeconomic shocks and macroeconomic fluctuations in production networks. Based on the model postulated by Acemoglu, Ozdaglar, and Tahbaz-Salehi (2010), the authors demonstrate that idiosyncratic shocks at the sectoral level may propagate and multiply in the network through the intersectoral relations of the productive system, generating non-negligible fluctuations in the macroeconomic aggregates. This propagation can be explained not only by the characterization of firms as direct suppliers, but also as indirect suppliers in the network's different linkages (the authors measure this aspect using the eigenvector centrality[28]). These higher order interrelations open the possibility of downstream cascade effects through which shocks are transmitted from buyer to buyer in the network. This way, the authors report two important findings for

[28] Specially, the higher the eigenvector centrality, the greater the likelihood that a highly central industry is related to other industries that are also highly central.

production networks with degree distributions that fulfill the power law. First, shocks on suppliers with a more central role in the network (i.e. a large number of direct and indirect buyers) have a higher impact on the volatility of macroeconomic aggregates. Second, the more heterogeneous the economic sectors' role as input suppliers is, the larger the effect of these shocks.[29] [30]

Jones (2013) examines the production network as a transmission and amplification channel of shocks generated by an inadequate resource allocation within firms. With this purpose in mind, he develops an equilibrium model for an open economy with a representative household and various economic sectors that exhibit Cobb-Douglas preferences and technologies, respectively. Within their production processes each sector employs capital, labor, and intermediate domestic and imported goods, and is dependent on the rest of the sectors' economic activity through input-supply relationships. With this model Jones finds that firm distortions (resulting from taxes, regulations, preferential credit rates) alter resource allocation and decrease the economy's total income. This effect depends on how important locally produced inputs are for economic sectors and how different these distortions turn out to be between sectors.

Carvalho et al. (2016) examine intersectoral relationships in the production network as a mechanism for the diffusion of microeconomic shocks, both downstream and upstream. Different from other research studies, their work is based on a general equilibrium model that involves competitive firms which use CES technologies. Based on this

[29] Gabaix (2011) finds a similar result when the firm size distribution fulfills the power law, without considering intersectoral relations in the network. This author demonstrates that as firm size becomes more heterogeneous (i.e. distribution tails are heavier), idiosyncratic shocks at the firm level are more likely to generate macroeconomic fluctuations. An introduction to the establishment of power laws in economics is carried out by Gabaix (2016).

[30] Carvalho (2014) shows the importance and scope of Acemoglu's results in the analysis of aggregate volatility, using data from the United States' production network. According to Carvalho, this network can be characterized as a small world (i.e. low network's diameter, low average distance), where economic sectors are found a few input-supply transactions from each other. This network is also characterized by a high heterogeneity in input provision (i.e. the power law holds in the degrees' distribution) and highly central economic sectors. These properties explain the dynamics of two interesting phenomena. First, the proximity between different economic sectors is highly correlated with the comovements of their economic activity. The closer they are, the higher is the probability that their activity occurs jointly. Second, the activity of the most central sectors in the US' production network is highly correlated with GDP growth. The latter suggests that sectors that are more central are an important source of aggregate fluctuations, since they enable the synchronization of the remaining sectors' activity in the network.

model, the authors report different findings which are dependent on the elasticity between intermediate consumption and labor demand. First, if a firm experiences a negative shock, production in all downstream firms decreases. This effect is intensified the higher the elasticity of substitution between inputs and labor. Second, if a firm experiences a negative shock, then production in all upstream firms decreases as well, but only when the elasticity is above 1 (otherwise, it increases). Third, both downstream and upstream effects on a firm in the production network decrease in magnitude the farther the firm is located from the firm that received the negative shock.[31]

Bigio and La'O (2016) study the diffusion of sectoral shocks which are financial in nature, and their effect on labor and the economy's total income. To this end, they use the multisectoral model by Acemoglu et al. (2012) and assume a representative household whose preferences include leisure and firms have technologies with decreasing returns to scale. Based on this model, they incorporate financial restrictions on the use of capital so that firms commit a fixed portion of their revenue for the acquisition of labor and capital goods within their production process. According to Bigio and La'O, this form of financing generates a gap between the firm's marginal revenue and cost which can be attributed to a rate of interest on the working capital. The model's results show that financial distortions may lead to an inefficient use of production factors and therefore, a reduction in the economy's total productivity. This effect is subject to which firm is affected by these distortions and how the production network's structure propagates them. Additionally, they demonstrate that these distortions preclude the satisfaction of Hulten's theorem[32], which usually holds in studies that analyze the diffusion of shocks in production networks in perfect markets.

Grassi (2017) also contributes to microeconomic level analyses of aggregate fluctuations and focuses on three aspects: firm size, the industrial organization of sectors and the production network that links them. For this, he builds an equilibrium

[31] Carvalho et al. (2016) use this model to quantify the direct and indirect effects following the 2011 earthquake in Japan. Specially, they found that the diffusion of the earthquake's effect was stronger on downstream firms in the production network and that this effect lessened as the length of the production linkages increased. Boehm, Flaaen, and Pandalai-Nayar (2019) carry out an analysis along the same line to measure the effect of the earthquake in Japan.

[32] Hulten's theorem states that total factor productivity growth can be explained by adding up sectorial shocks, weighted by the contribution of each sectors' sales to the economy.

model with imperfect competition that involves two agents: a representative household who demands goods and supplies labor and a set of firms distributed among the sectors that weave together the production network. These firms set their prices strategically à la Bertrand and use Cobb-Douglas technologies with constant returns to scale. It is assumed that each sector produces different varieties of one good and prices and quantities are established considering a CES type substitution. Based on this model, Grassi states that microeconomic shocks can generate the propagation of two effects: a downstream cascade effect due to price changes, and an upstream effect due to changes in profit margins. On the one hand, the size of the impacted firm and the intensity of competition in the sector to which it belongs govern the magnitude of sectoral changes in prices and profit margins, which results in both downstream and upstream cascade effects. On the other hand, the intensity of competition and centrality of the sector to which the impacted firm belongs determine only the scale of sectoral profit margins, which generates upstream cascade effects only.

Acemoglu, Ozdaglar, and Tahbaz-Salehi (2017) analyze the possibility of the emergence of strong fluctuations in macroeconomic aggregates (which they refer to as “macroeconomic tail risks”), using the general equilibrium model proposed by Acemoglu, Ozdaglar, and Tahbaz-Salehi. (2010). Based on this model, the authors report two main findings. First, the idiosyncratic shocks must follow a probability distribution with heavier tails than that of the normal distribution (e.g. the exponential tail distribution) for them to generate strong macroeconomic deviations and give rise to a crisis. Second, there needs to be enough heterogeneity in the size of the economic sectors so that the microeconomic shocks do not disappear following the aggregation process and so that they may generate a strong variation at the macroeconomic level. As a side effect, the authors observe that large deviations in the macroeconomic aggregates are accompanied by substantial and simultaneous variations in a wide range of industries, especially when intersectoral relationships in the production network are dense.

D. R. Baqaee (2018) examines how shocks at the sectoral level are diffused in the production network with non-competitive markets and external scale economies generated by firm entry and exit. His analysis uses an equilibrium model that includes a representative household and a continuum of homogeneous firms which belong to different industries, each using CES preferences and technologies, respectively. Here, firms set the selling price of the good they offer (without price discrimination) so that they attain a predetermined level of profit margins. The model's equilibrium is subject to the number of firms that participate in the industry, which is assumed to grow as product differentiation increases. According to this model, Baqaee finds that a productivity shock on an economic sector accentuates fluctuations in the macroeconomic aggregates since these shocks produce changes on prices, profits and consequently, on the number of firms that participate in each sector. The author demonstrates that the diffusion of this shock in the production network depends on the industry's role as an input supplier and buyer in the production network, and on the market structure as well. Specifically, he shows that these shocks may generate not only downstream cascade effects in the production network, but upstream cascade effect as well, when markets are non-competitive, and elasticities of substitution are different from one (i.e. technological specifications are different from Cobb-Douglas).

Finally, Baqaee and Farhi (2019) examine the diffusion of microeconomic shocks and their impact at the macroeconomic level, as a non-lineal extension of Hulten's theorem. The authors use a simple equilibrium model with one representative household and various production sectors in competitive markets. Different from other models developed in this field of study, the production sectors use technologies with constant elasticities of substitution for every pair of inputs. Within this framework, the authors find that trade relations in the production network are not the only key factors in the amplification of negative microeconomic shocks and the mitigation of distortions caused by positive shocks; nonlinearities that stem from the elasticities of substitution (and returns to scale and factor reallocation as well) are also crucial. Specially, it is demonstrated that nonlinearities create second order effects that are non-negligible for the macroeconomic aggregates, which is why the economic growth's variance may be larger than that obtained when using Cobb-Douglas technologies. But in addition, the economic growth's distribution is biased to the left, with heavier tails, even when microeconomic shocks have a symmetric and mesokurtic distribution.

The studies referenced previously exhibit a common theoretical finding: the production network constitutes a key economic mechanism to understand how microeconomic shocks are propagated in the economic system and generate nontrivial fluctuations in the macroeconomic aggregates.

As can be seen from the literature, this result has a strong theoretical foundation in the development of general equilibrium Walrasian models (with the exception Grassi (2017) and D. R. Baqaee (2018)). In assuming perfect competition, this type of models overlook an important market failure that characterizes the productive systems in reality: price setting. In general terms, academic works on the diffusion of economic shocks in production networks consider price-taking firms, irrespective of the network's topology. Here, the mechanism that determines selling prices is always the equilibrium, regardless of the extent of a firm's intermediation and its participation in the productive network. These assumptions limit the possibility of studying oligopoly situations in which firms interact during price formation, considering each firm's linkages and the level of rivalry present in their industry. These deviations from the perfect markets assumption may accentuate the effect of any shock in the network (depending on the size of the industry[33]) and impact other key economic variables. For example, a productivity shock that affects a large industry may generate not only an output reduction in its surrounding firms, but also, an inflationary cascade effect due to the increase in its marginal cost. Other market failures also deserve attention like, for example, disequilibrium and incomplete markets.

Another distinctive aspect of equilibrium models in the study of the propagation of microeconomic shocks is the type of technology used to describe the economic sectors' production process. Here, studies generally assume a production function with a Cobb-Douglas specification and constant returns to scale. As it is known, demand functions that maximize profits under this technological restriction are lineal in nominal terms, meaning, input expenditure is a fixed share of the revenue generated by a sector. This characteristic may weaken the diffusion of economic shocks in the production network and reduce variations in macroeconomic fluctuations. For example, when considering more complex technologies, with elasticities of substitution different from one, sunk

[33] One of the traditional results of industrial organization theory states that as the number of firms within an oligopoly rises, the prices that firms set converge to the Walrasian equilibrium price (Cournot model). This property is based on homogeneous industries where there are no entry barriers, which is why it is expected that the analysis of the diffusion of microeconomic shocks under the assumption of competitive markets constitutes a valid approach only for these situations.

costs and increasing or diminishing returns to scale, or with the possibility of inaction, one of the shocks may propagate in a different way (downstream or upstream) or in an accelerated manner. A sample of that can be found in the results reported by Carvalho et al. (2016), Grassi (2017) and Baqaee and Farhi (2019), when they assume CES type production functions.

A critical aspect that is absent from the analysis of the economic shock diffusion is firm bankruptcy. In general, firm bankruptcy as a phenomenon is omitted from the analysis of shock diffusion because of the representative firm assumption that is included in general equilibrium models. This assumption leads to the dismissal of differences in firms' technology, and as such, the possibility of identifying those firms that may exit the industry due to circumstances inherent to their economic activity. Consequently, the supply functions of equilibrium models usually adopt a smooth and continuous form (i.e. well-behaved functions), therefore precluding the emergence of any drastic change in the economic flows. This fact suppresses an important source of variation in the determination of macroeconomic aggregates, and of course, in the way in which shocks are propagated in the production network. For example, if a shock is sufficiently strong to cause a large firm goes bankrupt, this will be followed by a significant change in the supply curve, which, depending on the linkages of the impacted firm, will be diffused among suppliers and buyers in the network, and possibly endanger their permanence in the economy. In other words, an economic shock may trigger a systemic bankruptcy process (downstream or upstream) in the network and generate sizable fluctuations in the macroeconomic aggregates. D. R. Baqaee (2018) makes some contributions to the study of how firm entry and exit predetermine the impact of these shocks; however, like the other studies, his analysis is carried out at the sectorial level with a continuum of homogeneous firms.

Because of interrelations in the production network and firm heterogeneity, it is advisable to study a firm's bankruptcy using a random approach, which goes beyond the deterministic approach with which this phenomenon has been generally theorized.[34]

---

[34] In the literature, entry and exit decisions have been analyzed essentially using a deterministic choice approach, following guidelines established by Marshallian rules (i.e. the decision to produce depends on whether the value of revenue exceeds certain determined thresholds). There exists a wide range of works that analyze these decisions in a partial equilibrium context, considering uncertainty in the short run,

Incorporating the assumption of firm bankruptcy as a random event brings about two advantages for the economic analysis. First, it makes it possible to address the high complexity and unpredictability of the productive system when it is comprised by a large number of firms that act in a simultaneous and interdependent manner in a production network. Second, the random approach enables the consideration of non-observable factors within a firm (circumstances that are political, organizational or institutional in nature) which condition its economic activity and its potential exit from the market.

Finally, the theoretical analysis of the diffusion of economic shocks has centered its attention on exogeneous production networks, in other words, networks with input-supply relations that do not dependent on firm behavior neither market prices. This assumption severely limits the propagation of shocks in a network, since it discards another important source of variability in the fluctuations of macroeconomic aggregates: the endogenous formation of market transactions. As such, the propagation of a shock not only depends on the degree of the firms' interrelations in the production network, but also on the circumstances under which the input-supply relations arise in the production network. A shock may diffuse and multiply in the network to the extent to which those interrelations are strong enough in the face of the market pressures, and of course, the strategies adopted by other firms in the production network. Additionally, the propagation of shocks in the network will grow in complexity depending on how these relationships are formed (that is to say, if they are deterministic or stochastic) and depending on the variables that explain this formation (e.g. prices, information asymmetry, rivalry, organization, etc.).

2.3 Model

In this section we develop a model for the analysis of a production network with imperfect competition and firm bankruptcy. The economic system is comprised by a representative household h and firms $J = \{1,2,\dots,N\}$ both of which transact in the market for goods $I = \{1,2,\dots,M\}$ and the labor market l. It is assumed that $N > M$.

---

investment and market structure. Some known examples include the work of Sandmo (1971), Dixit (1989), Lambrecht (2001) and Kwon (2010)

The production network is defined by a directed graph $R = (V, \Gamma)$, where $V = J \cup \{h\}$ are the nodes that denote the set of firms J and the representative household h, and $\Gamma \subset V \times V$ are the edges that represent the buying-selling transactions that agents carry out in the goods market I and the labor market l. It is worth noticing that this is a potential network, meaning it comprises all possible buying-selling transactions between the firms.

Each firm $j \epsilon J$ produces only one good, denoted by the set $a(j)$, and demands a set of inputs which are denoted by the set $b(j)$. Undoubtedly, $a(j) \subset I, |a(j)| = 1$ and $b(j) \subset I \cup \{l\}$. It is assumed that firms do not use their own produced goods as inputs in their production processes and that they always require labor $\left(i\text{.e. } a(j) \cap b(j) = \emptyset, l \in b(j)\right)$. The firms' technology assumes free disposal, no free lunch, a closed set and includes the possibility of inaction. On the other hand, the representative household h supplies labor $a(h) = \{l\}$ and demands a set of goods $b(h) \subset I,\ b(h) \neq \emptyset$. Household preferences are assumed to be rational, continuous, locally non-satisfied and strictly convex.[35]

Correspondences between $a(j)$ and $b(j)$ are defined in such a way so that all goods have at least one supplier and one buyer.

$$\forall i \in I \cup \{l\}, \exists j, j' \in V, j \neq j' \mid \{i\} = a(j)\ \wedge\ i \in b(j') \qquad (1)$$

These correspondences assume that firms are exclusively dedicated to the production of one type of good, be this an intermediate or final good, but not both. In formal terms, let $I^{\mathfrak{m}} = I \backslash b(h)$ be the set of intermediate consumption goods with $M^{\mathfrak{m}}$ cardinality, and $I^{\mathfrak{g}} = b(h)$ the set of final consumption goods with $M^{\mathfrak{g}}$ cardinality. Likewise, let $J^{\mathfrak{m}} = \{j \in J \mid a(j) \subset I^{\mathfrak{m}}\}$ be the set of firms that produce intermediate goods with $N^{\mathfrak{m}}$ cardinality, and $J^{\mathfrak{g}} = \{j \in J \mid a(j) \subset I^{\mathfrak{g}}\}$ the set of firms that produce final goods with cardinality $N^{\mathfrak{g}}$. This way, the following proposition holds:

$$J^{\mathfrak{m}} \cap J^{\mathfrak{g}} = \emptyset, J^{\mathfrak{m}} \cup J^{\mathfrak{g}} = J$$

---

[35] For the simulations' purposes, technologies and preferences are represented by CES functions.

Let $c(k,i)$ be the set of potential suppliers of the intermediate good $i \in b(k)\backslash\{l\}$ for the firm $k \in J$. This set, for every firm and for every input different from labor is assumed to be comprised by two potential suppliers $c(k,i) = \{j, \bar{j}\}$, where j is the regular supplier, meaning, the firm that usually trade with buyer k; and $\bar{j}$ is the rival supplier, in other words, the firm that has never before engaged in a transaction with the buyer k but which tries to replace supplier j.

In formal terms, the following condition must hold for the set $c(k,i)$:

$$\forall k \in J, \forall i \in b(k), \exists c(k,i) = \{j, \bar{j}\} \subset J^{\mathrm{m}}, j \neq \bar{j} \mid a(\bar{j}) = a(j) = \{i\}$$

The correspondences between $a(j)$, $b(j)$, $c(k,i)$ make it possible to define the structure of the production network. Here, the edge (j,k) is part of the network $R = (V, \Gamma)$ in any of the following cases: the representative household h supplies labor to the firms $k \in J$; firm $j \in J$ produces a final consumption good for the representative household $k = h$; or, firm j is a supplier (regular or rival) for the firm k. These conditions may be expressed in the following way:

$$\begin{gathered} \forall j \in J, \qquad a(h) \cap b(j) \neq \emptyset \Rightarrow (h,j) \in \Gamma \\ \forall j \in J, \qquad a(j) \cap b(h) \neq \emptyset \Rightarrow (j,h) \in \Gamma \\ \forall j,k \in J, j \neq k : \exists i \in b(k) \mid j \in c(k,i) \Rightarrow (j,k) \in \Gamma \end{gathered}$$

In this network, firm behavior as input buyers is modelled using a probabilistic choice approach. Each firm $k \in J$ assigns a probability ψ_{jk} to the event of successfully assigning a contract for the provision of good i to supplier $j \in c(k,i)$. This behavior stems from the buyer's lack of information with respect to its suppliers (regarding aspects such as the quality of the product they offer, delivery times, specific and required input characteristics, etc.), and the trade-off between expected profits and the risk faced by the buyer.

Here, transactions with the regular supplier j generate a lower risk for buyer k, since the recurring relationship with this supplier has enabled the buyer to eliminate some sources of risk (for example, adapting its productive processes to the characteristics of the regular supplier). On the other hand, transactions with the rival supplier $\bar{j}$ are riskier, essentially because of the lack of information. In this sense, to be competitive, the rival supplier must offer a sufficiently low price so that buyer k's expected profit is higher than that offered by the regular supplier.

Therefore, the probabilities ψ_{jk} depend on the selling prices set by input suppliers in the production network. Let $p^{\mathrm{m}} = \left(p_{jk}^{\mathrm{m}}\right) \in \mathbb{R}_+^N \times \mathbb{R}_+^{N^{\mathrm{m}}}$, the prices set by intermediate good firms, where p_{jk}^{m} is the price offered by supplier j to firm k, when $j \in c(k,i), a(j) = \{i\}$. Then, the probability ψ_{jk} of buyer k transacting with its usual supplier j is determined by the price this supplier offers, p_{jk}^{m}, and the price offered by its rival, $p_{\bar{j}k}^{\mathrm{m}}$:

$$\forall k \in J, \forall i \in b(k), \qquad \psi_{jk} = \psi_{jk}\left(p_{jk}^{\mathrm{m}}, p_{\bar{j}k}^{\mathrm{m}}\right) \;\; j, \bar{j} \in c(k,i)$$

These functions can be used to define the probabilistic space (Ω, Ψ), where $\Omega = \left\{\omega = \left(V, \hat{\Gamma}\right) \mid \hat{\Gamma} \subseteq \Gamma \right\}$ is the set of realizations in the production network $R = (V, \Gamma)$. Here, the edges $(j,k) \in \hat{\Gamma}$ if firm k effectively assigns the input provision contract to firm j.[36] On the other hand, $\Psi(\omega \mid p^{\mathrm{m}})$ is the probability of the realization of $\omega \in \Omega$, given prices p^{m} and considering risks as given.

The set of realizations Ω assumes that every firm may offer its produced good to one or more buyers (or to none), and that it may demand any input from only one supplier. Formally, let $\hat{\Gamma}(\omega)$ be the buying and selling transactions of realization $\omega \in \Omega$. Let $\Delta_j^+(\omega) = \{k \epsilon V \mid (j,k) \in \hat{\Gamma}(\omega)\}$ and $\Delta_{ji}^-(\omega) = \{k \epsilon V \mid (k,j) \in \hat{\Gamma}(\omega), a(k) = \{i\}\}$, be two operators that show the buyers and suppliers respectively of firm j in each realization. Then, for the set Ω the following proposition holds:

$$\forall \omega \in \Omega, \forall j \in J, \forall i \in I \quad \left|\Delta_j^+(\omega)\right| \geq 0, \qquad \left|\Delta_{ji}^-(\omega)\right| = 1$$

Likewise, it is assumed that all transactions between firms J and the representative household h are effective in any realization $\omega \in \Omega$, which is why for the set Ω the following proposition also holds:

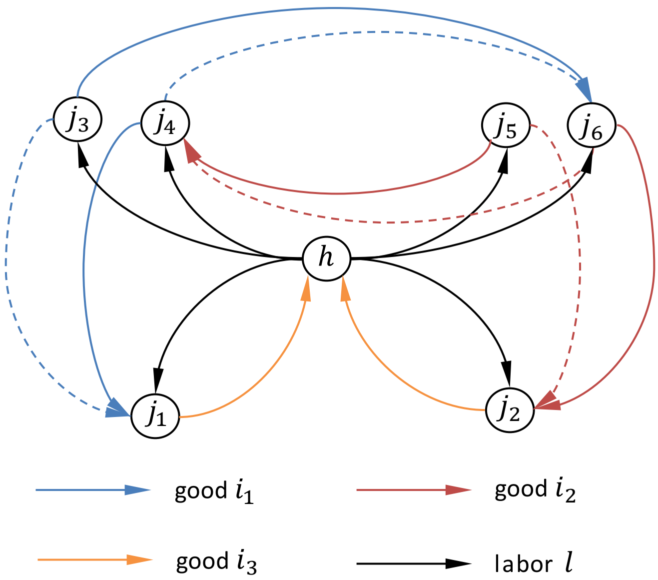
$$\forall \omega \in \Omega, \forall j \in J \mid (a(j) \cap b(h) \neq \emptyset \vee a(h) \cap b(j) \neq \emptyset), \qquad \{(j,h),(h,j)\} \subset \hat{\Gamma}(\omega)$$

To appreciate the model's formulation, let's consider the production network shown in Figure 2.1. In this network, there are 6 firms $J = \{j_1, j_2, j_3, j_4, j_5, j_6\}$ and a representative household h, which interact in 3 goods markets $I = \{i_1, i_2, i_3\}$ and the labor market l.

---

[36] It is worth noting that for each realization $\omega \in \Omega$ denotes a different set of effective transactions for the firms, meaning it denotes a subgraph of R.

Figure 2.1. An example of Production Network R



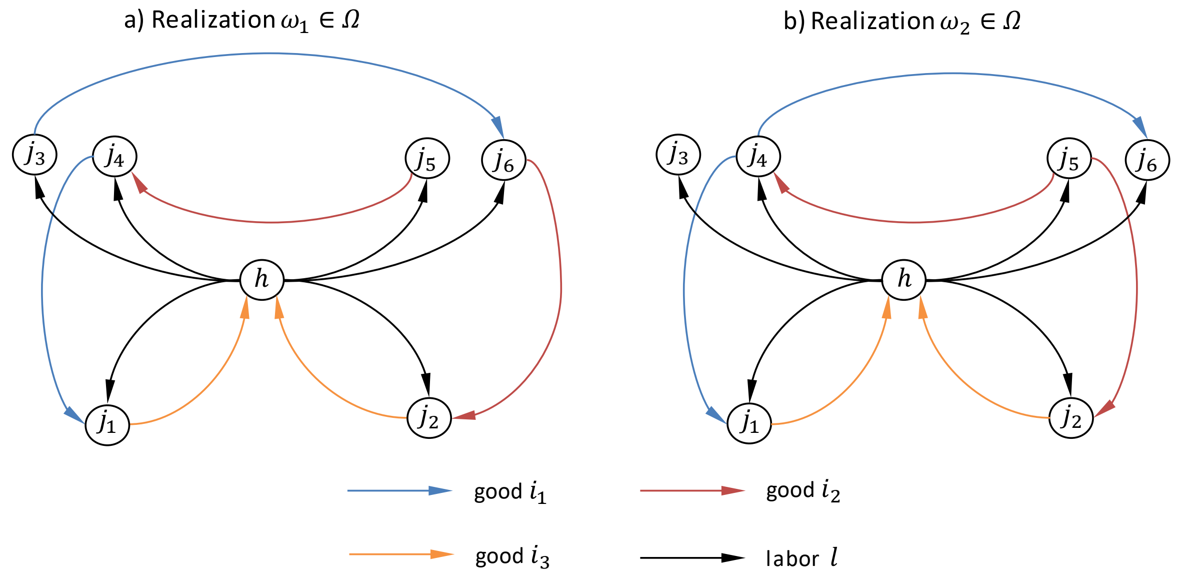
Note. This figure illustrates an example of a production network. Each node denotes a firm, each edge denotes a potential buying and selling transaction, and each color identifies a different good. If the edges are drawn with a continuous line, they represent the supply from the regular provider; if the edges are drawn with a dotted line, they denote the supply from the rival supplier.

Firm output is defined by the products $a(j_1) = \{i_3\}$, $a(j_2) = \{i_3\}$, $a(j_3) = \{i_1\}$, $a(j_4) = \{i_1\}$, $a(j_5) = \{i_2\}, a(j_6) = \{i_2\}$; while their demand for inputs is defined by the sets $b(j_1) = \{i_1, l\}$, $b(j_2) = \{i_2, l\}$, $b(j_3) = \{l\}$, $b(j_4) = \{i_2, l\}$, $b(j_5) = \{l\}$, $b(j_6) = \{i_1, l\}$. On the other hand, the representative household's preferences are defined over the items $a(h) = \{l\}$ y $b(h) = \{j_1, j_2\}$.

Transactions are shown in different colors. Transactions corresponding to good i_1 are drawn in blue, transactions involving good i_2 , in red, transactions corresponding to good i_3, in orange and labor transactions are drawn in black. As can be observed, the markets $I^{\mathrm{m}} = \{i_1, i_2\}$ are markets for intermediate goods where firms $J^{\mathrm{m}} = \{j_3, j_4, j_5, j_6\}$ trade; meanwhile, market $I^{\mathrm{g}} = \{i_3\}$ is the market for final goods, where firms $J^{\mathrm{g}} = \{j_1, j_2\}$ trade.

Regular and rival suppliers for each firm are distinguished by continuous or dotted edges, respectively. For example, firm j_2's regular supplier of good i_2 is firm j_6 and its rival supplier is firm j_5 (i.e. $c(j_2, i_1) = \{j_6\ , \bar{j}_5\}$). Similarly, firm j_6's regular supplier of good i_1 is firm j_3 and its rival supplier is firm j_4 (i.e. $c(j_6, i_1) = \{j_3\ , \bar{j}_4\}$).
Two realizations in this production network are shown in Figure 2.2.

Figure 2.2. Examples of realizations in the production network



Note. This figure shows two realizations in the production network of Figure 2.1. Figure 2.2.a displays transactions that correspond exclusively to regular suppliers, while Figure 2.2.b shows those that pertain to some rival suppliers.

In realization ω_1 all firms acquire their inputs from their corresponding regular suppliers. For example, firm j_2 chooses to purchase from its regular supplier j_6 (i.e. $(j_6, j_2) \in \omega_1$), and firm j_6 decides to purchase from its regular supplier j_3(i.e.($(j_3, j_6) \in \omega_1$). On the other hand, in realization ω_2 some buyers engage in transactions with their rival suppliers. Here, firm j_2 decides to transact with its rival supplier j_5 (i.e. $(j_5, j_2) \in \omega_2$) and firm j_6 chooses to buy from its rival supplier j_4 (i.e. $(j_4, j_6) \in \omega_2$). In this realization it is worth emphasizing that the buyers' decisions enable firms j_4 y j_5 to monopolize the market for goods i_1 y i_2, respectively.

Given the possible realizations Ω in the production network $R = (V, \Gamma)$, all firms and the representative household make their best decision. Intermediate good firms maximize their expected profits by setting their output prices, given the network's structure and the chances of selling their output to their buyers. Final good firms also maximize their expected profits but in competitive markets, which is to say they do not set their prices but their produced quantities since they are price takers. Finally, the representative household maximizes its expected utility through its demand for final goods and labor supply.

Once a production network's realization has been carried out based on this behavior, it is necessary to ensure the supply and demand equilibrium for each and every market, including the labor market. To do so, wages and prices of final consumption goods must be adjusted *ex post* in such a way that all markets clear. We name the set of all prices and quantities in this equilibrium the General Market Equilibrium (GME).

Next, the microeconomic foundation of the process of probabilistic choice that corresponds to the buyer will be explained, before describing the economic agents' behavior in the model.

2.3.1 The buyer's probabilistic choice problem.

2.3.1.1 Formulation

As mentioned in the previous section, the buyers' selection between one supplier or another is probabilistic; that is, a buyer assigns a probability to each of the transactions it engages in with its potential suppliers.

This kind of selection arises from the buyer's lack of information concerning key aspects of the transaction, such as: product's quality, delivery time, paying or financing system, customer service, response capabilities, transportation costs, among other factors. When the buyer interacts repeatedly with a supplier, it acquires knowledge about these factors and even adapts its production process to the characteristics of the input provided by this supplier. Therefore, a supplier that has engaged in transactions previously and in a regular manner with a firm (in other words, a regular supplier) has an advantage over a supplier which has not (a rival supplier), even when the competing price is lower, granting the former a higher probability of selling its product. Naturally, if the rival supplier's price is much lower than that of the regular supplier, the buyer might be interested in switching suppliers; however, it would face the risk of the input not fulfilling all the required characteristics for its production process, introducing the possibility of incurring in losses instead of profits. [37]

[37] It is necessary to underscore that the basis of probabilistic choice assumes incomplete contracts in input provision; in other words, contracts that cannot guarantee that the supplier fulfills all terms and conditions required by the buyer regarding the input's transaction (due to the complexity, uncertainty or informality in the transaction). In cases where the contract turns out to be complete, the basis of probabilistic choice is no longer appropriate since the buyer would receive its inputs with all required specifications satisfied, or

It is important to underscore that the problem of probabilistic choice presented here is different from the conventional risk aversion model, when an agent makes a deterministic decision that maximizes its expected utility. It also differs from other alternative frameworks like models of loss aversion, probability weighting and models of context-dependence (Donoghue and Somerville 2018).[38]

In the probabilistic choice problem, the buyer assigns a probability to a transaction with a supplier, instead of making a discrete decision of which supplier it will engage in a transaction with. This probability determines the states of nature that are generated because of the buyer's lack of information concerning its suppliers.[39]

Next, this decision is modelled. Let's suppose there is a buyer $k \in J$ and two suppliers: a regular supplier j and a rival supplier $\bar{j}$ which provide the same good $a(j) = a(\bar{j})$, as shown in Figure 2.3. Both suppliers operate in two kind of scenarios: a positive scenario where they generate profits for the buyer, and a negative scenario where they generate losses.

This way, if the buyer decides to engage in a transaction with its regular supplier j, then its expected profits will be $\Pi_j = \left(1 - \phi_j\right)\pi_j + \phi_j \varepsilon_j$, where π_j, ε_j are the level of profits and losses obtained with this supplier, respectively, and ϕ_j will be the risk of incurring losses. Similarly, if the buyer decides to engage in a transaction with the rival supplier $\bar{j}$, then its expected benefits will be $\Pi_{\bar{j}} = \left(1 - \phi_{\bar{j}}\right)\pi_{\bar{j}} + \phi_{\bar{j}}\varepsilon_{\bar{j}}$, where $\pi_{\bar{j}}, \varepsilon_{\bar{j}}$ are the level of profits and losses obtained with this supplier, respectively, and $\phi_{\bar{j}}$ is the risk of incurring losses. In both cases, losses ε_j and $\varepsilon_{\bar{j}}$ may be so high as to be considered catastrophic magnitudes $\left(\varepsilon_j, \varepsilon_{\bar{j}} \ll 0\right)$.

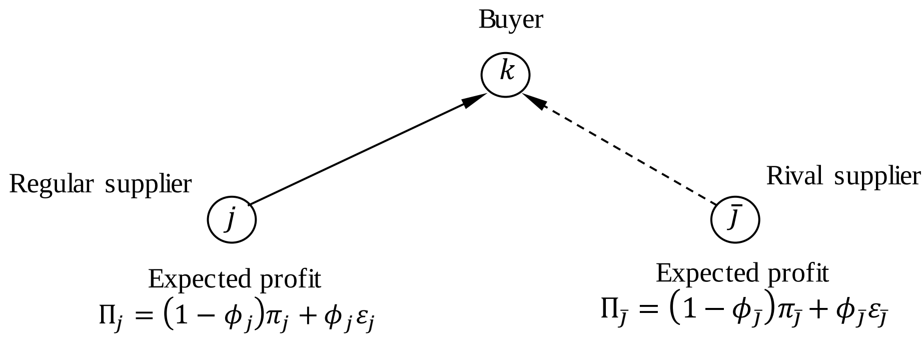
---

it would receive assurances where it the case that it was not satisfied with the transaction, and this way, it would always safeguard its profits. We thank Felipe Brugués for this observation.

[38] Donoghue and Somerville (2018) compare the results of these models in the fields of insurance, financial investments and agency problems.

[39] The theory of quantum choice is one of the first analysis that considers probability as a variable of choice in the behavior of economic agents. This theory considers that individuals may choose between a set of discrete options on the basis of a random utility model. In this model, the agent first chooses randomly its utility function and afterwards, it chooses the utility maximizing alternative. Mcfadden (1976) carries out a brief review of the results of this model. However, up to what is known, there is no empirical application as to why the agent would make its decisions in this manner.

Figure 2.3. Probabilistic choice between two suppliers



	Value ; Probability	Value ; Probability
Positive scenario	$\pi_j > 0$; $(1-\phi_j)$	$\pi_{\bar{j}} > 0$; $(1-\phi_{\bar{j}})$
Negative scenario	$\varepsilon_j < 0$; ϕ_j	$\varepsilon_{\bar{j}} < 0$; $\phi_{\bar{j}}$

Note. The figure above illustrates the formulation of a buyer's probabilistic choice between two suppliers. Node k denotes the buyer, while nodes j and $\bar{j}$ denote the regular and rival suppliers, respectively. The table below shows the buyer's benefits and probability of engaging in a transaction with each different supplier, under two scenarios: a positive scenario where there are profits and a negative one where there are losses.

Let's suppose a transaction with the rival supplier $\bar{j}$ generates a larger expected profit than a transaction with the regular supplier j; however, it would still involve a larger expected loss.

$$\Pi_{\bar{j}} > \Pi_j, \qquad \phi_{\bar{j}}\varepsilon_{\bar{j}} < \phi_j\varepsilon_j$$

These aspects state a dilemma for the buyer. Which transaction should the buyer decide to engage in? The one that awards the larger profit or the one that poses the lower risk? If the buyer is only interested in maximizing its profits, then it will choose to engage in a transaction with the rival supplier $\bar{j}$; but, if the buyer is also interested in minimizing risk, it will choose to transact with the regular supplier j.

If the buyer simultaneously weights the variations in the expected benefits against the variations in the risk involved in hiring a specific supplier, its decision will not necessarily be a deterministic one. Let ψ_{jk} be the probability the buyer assigns to transacting with the regular supplier j, and $\psi_{\bar{j}k} = (1-\psi_{jk})$, the probability the buyer assigns to transacting with the rival supplier $\bar{j}$ (i.e. non-regular supplier). Let g be a function that denotes the trade-off between the variation in expected profits and risk for the specific probability ψ_{jk}. We state that in weighing this trade-off, the buyer wants a minimum value, that is to say:

$$g\left(\widehat{\Pi}(\psi_{jk}), \hat{\mathcal{E}}(\psi_{jk})\right) \geq \beta$$

Where $\widehat{\Pi}(\psi_{jk}) = (1-\psi_{jk})(\Pi_{\bar{j}} - \Pi_j)$ is the expected profits differential, $\hat{\mathcal{E}}(\psi_{jk}) = (1-\psi_{jk})(\phi_{\bar{j}}\varepsilon_{\bar{j}} - \phi_j\varepsilon_j)$ is the expected losses differential and β is the minimum preference value for the trade-off. [40]

It is assumed that g is a continuous, increasing and concave function ($g_1 \geq 0, g_2 \geq 0, g_{11} \leq 0, g_{22} \leq 0, g_{12} \geq 0$). For the sake of simplicity, this function is represented using a univariate function $f(\psi_{jk})$:

$$f(\psi_{jk}) = g\left(\widehat{\Pi}(\psi_{jk}), \hat{\mathcal{E}}(\psi_{jk})\right)$$

In this context, the problem for buyer k involves finding the probability that maximizes expected profits from engaging in a transaction with suppliers $j, \bar{j}$; subject to the restriction that stems from the trade-off between profits and risk.

$$\begin{aligned}
&\max_{\psi_{jk}} \psi_{jk}\Pi_j + (1-\psi_{jk})\Pi_{\bar{j}} \\
&s.r \\
&f(\psi_{jk}) \geq \beta \\
&0 \leq \psi_{jk} \leq 1
\end{aligned} \tag{2}$$

The solution to this problem is presented in Figure 2.4, which shows a situation where $\Pi_{\bar{j}} > \Pi_j$ y $\phi_{\bar{j}}\varepsilon_{\bar{j}} < \phi_j\varepsilon_j$.. For the existence of an interior solution, it is assumed that $f(1) > \beta, f(0) < \beta, f'(\psi_{jk}) \geq 0$.[41] The AB curve represents the function $f(\psi_{jk})$ and the straight line CD represents the horizontal which corresponds to a specific level of β in the y-axis. Both lines meet at point M and delineate the feasible set $f(\psi_{jk}) \geq \beta$, represented by the EF segment. On the other hand, the red line denotes the function for expected profit, $\psi_{jk}\Pi_j + (1-\psi_{jk})\Pi_{\bar{j}}$. In this way, as can be observed from Figure 2.4,

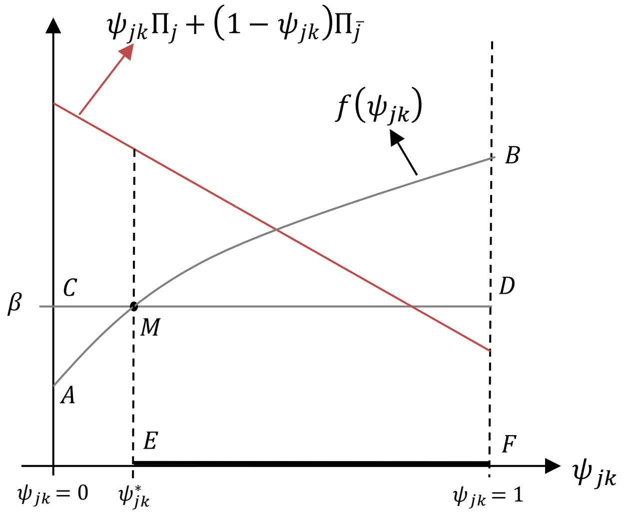
---

[40] The definition of the trade-off function g and the minimum value β may cause some ambiguity within the framework of microeconomic theory. Appendix C presents a reformulation of the problem of probabilistic choice that is consistent with the theory of preferences, without using this function.

[41] This implies $\frac{g_1}{g_2} \leq -\frac{\phi_{\bar{j}}\varepsilon_{\bar{j}} - \phi_j\varepsilon_j}{\Pi_{\bar{j}} - \Pi_j}$

the solution to problem (2) is reached at point E. At this point, the largest expected value is obtained while restricting probability ψ_{jk}^* to the trade-off between risk and profit.

Figure 2.4. Solution to the problem of probabilistic choice



Note. This figure shows the solution to the problem of probabilistic choice (2) when $\Pi_{\bar{j}} > \Pi_j$ y $\phi_{\bar{j}}\varepsilon_{\bar{j}} < \phi_j\varepsilon_j$. The abscissa axis denotes the probability of transaction and the ordinate axis denotes the total expected profit. The AB curve represents the trade-off function $f(\psi_{jk})$, the CD line is a horizontal that denotes the minimum preferred value β and the EF straight line denotes the problem's feasible set. The red line represents the expected profit function $\psi_{jk}\Pi_j + (1-\psi_{jk})\Pi_{\bar{j}}$ that is to be maximized and which reaches its optimal level at point E.

Let's perform a static comparative analysis of this solution when the level of profit varies. To do so, we calculate the derivative of probability ψ_{jk}^* with respect to profits Π_j and $\Pi_{\bar{j}}$. Using the implicit derivative theorem yields:

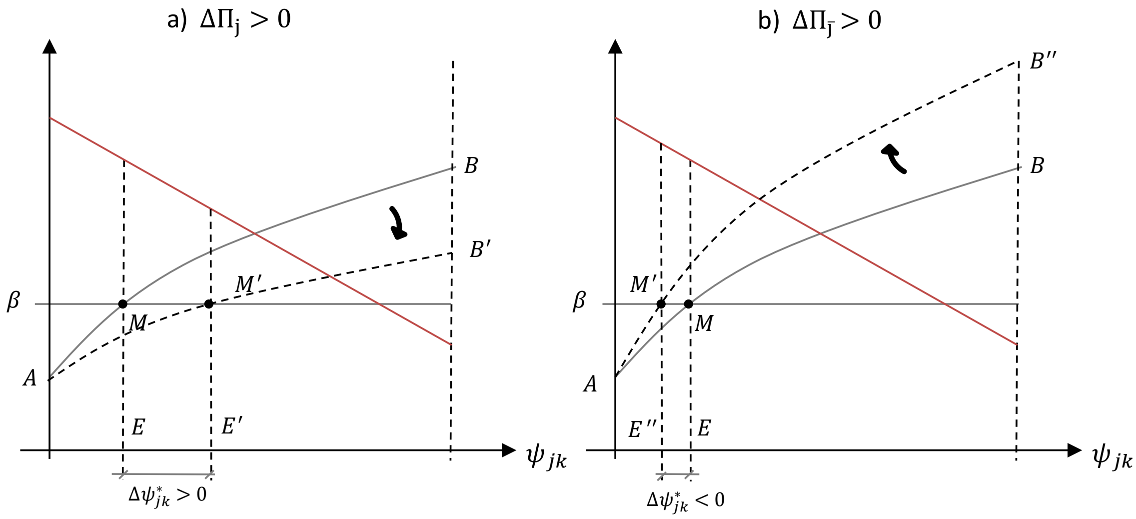
$$\frac{\partial\psi^*}{\partial\Pi_j} = \frac{g_1(1-\psi_{jk}^*)}{f'} > 0$$

$$\frac{\partial\psi^*}{\partial\Pi_{\bar{j}}} = -\frac{g_1(1-\psi_{jk}^*)}{f'} < 0$$

Both effects have the anticipated signs. If the expected profit Π_j rises (holding everything else constant), then the buyer will increase its probability of engaging in a transaction with the regular supplier j, since this supplier provides greater opportunities for increasing its profits. On the contrary, if the expected profit $\Pi_{\bar{j}}$ increases, then the buyer will have a higher preference for transacting with the rival supplier $\bar{j}$ and will decrease its probability of engaging in a transaction with the regular supplier j.[42]

These effects are illustrated visually in Figure 2.5. In graph 5a, the rise in profit Π_j contracts curve AB down to curve $A'B'$ (since $\frac{\partial \hat{\Pi}}{\partial \Pi_j} < 0$), which increases probability ψ_{jk} of engaging in a transaction with the regular supplier j. On the other hand, in graph 5b, an increase in profits $\Pi_{\bar{j}}$ expands curve AB up to curve $A''B''$ (since $\frac{\partial \hat{\Pi}}{\partial \Pi_{\bar{j}}} > 0$), which decreases probability ψ_{jk} of engaging in a transaction with the regular supplier j (which is the same as saying that probability $\psi_{\bar{j}k} = (1 - \psi_{jk})$ of engaging in a transaction with the rival supplier $\bar{j}$ increases).

Figure 2.5. Comparative statics of the problem of probabilistic choice



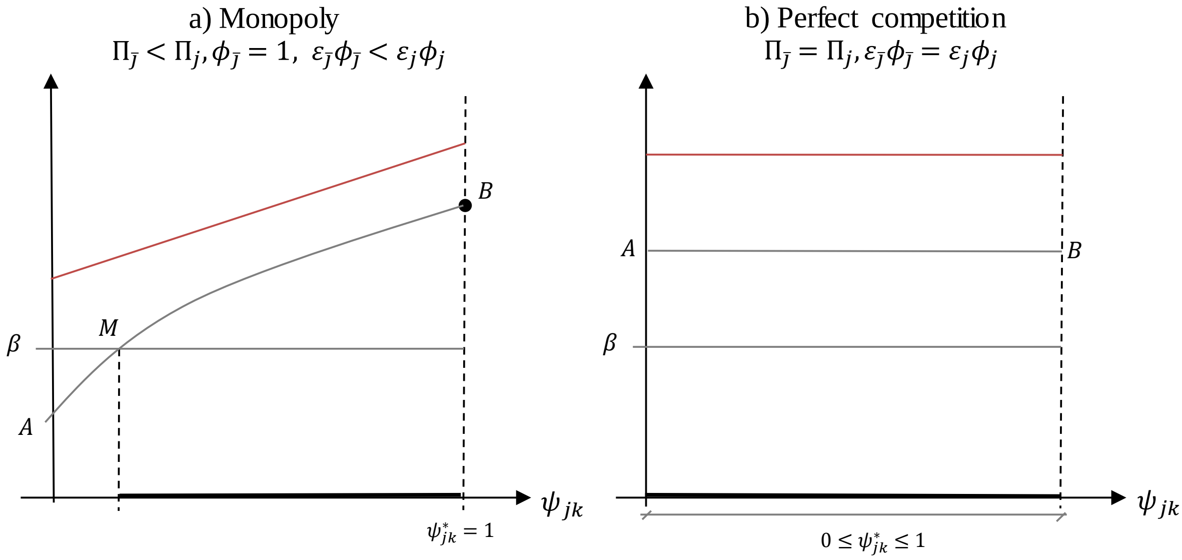
Note. This figure shows changes in the buyer's probabilistic choice (2) under two scenarios. Figure a) shows the resulting change in the probability of transaction ψ when the expected profit Π_j rises. On the other hand, figure b) shows the resulting variation in the probability of transaction when the expected profit $\Pi_{\bar{j}}$ increases. In both situations it is assumed that $\Pi_{\bar{j}} > \Pi_j$ y $\phi_{\bar{j}}\varepsilon_{\bar{j}} < \phi_j\varepsilon_j$. For further detail, refer to the description of Figure 2.4 in the text.

[42] **A**nother intuitive result is $\frac{\partial \psi^*}{\partial \phi_j} < 0, \frac{\partial \psi^*}{\partial \phi_{\bar{j}}} > 0$. This means that the buyer will increase its probability of transacting with usual supplier j if it reduces the risk of incurring loses or the rival supplier $\bar{j}$ increases its corresponding risk.

2.3.1.2 An approach to monopoly and perfect competition

One of the advantages of the probabilistic choice problem (2) is the formulation of different market structures such as monopoly and perfect competition. These settings are illustrated in Figure 2.6.

Figure 2.6. Extreme cases of the problem of probabilistic choice



Note. This figure shows the solution to the problem of probabilistic choice (2) under two market structures. Figure a) shows the probability of transaction when the market structure is a monopoly, that is when $\Pi_{\bar{j}} < \Pi_j$, $\phi_{\bar{j}} = 1$, and $\varepsilon_{\bar{j}}\phi_{\bar{j}} < \varepsilon_j\phi_j$. On the other hand, figure b) shows the probability of transaction under perfect competition, that is, when $\Pi_{\bar{j}} = \Pi_j$ and $\varepsilon_{\bar{j}}\phi_{\bar{j}} = \varepsilon_j\phi_j$. For further detail, refer to the description of Figure 2.4 in the text.

On the one hand, let's suppose that transactions with the rival supplier $\bar{j}$ involve absolute risk, in other words, $\phi_{\bar{j}} = 1$, which would mean $\Pi_{\bar{j}} = \varepsilon_{\bar{j}} < 0$, and therefore, $\Pi_{\bar{j}} < \Pi_j$ and $\phi_{\bar{j}}\varepsilon_{\bar{j}} < \phi_j\varepsilon_j$. This scenario is shown in Figure 2.6.a. In this case, the optimal solution to problem (2) is found at $\psi^*_{jk} = 1$ because the slope of the straight line representing the expected profit $\psi_{jk}\Pi_j + (1 - \psi_{jk})\Pi_{\bar{j}}$ becomes positive. This behavior grants a monopolistic nature to the regular supplier j, since for the buyer, engaging in a transaction with the rival supplier $\bar{j}$ would always incur in great losses.

Of course, this is an extreme case. However, for there to be a monopoly, the buyer' evaluation of risk need only be high enough so that $f(\psi_{jk}) > \beta$ always holds or, the expected profit when engaging in a transaction with the rival supplier must always be below that of transacting with the regular supplier.

On the other hand, let's suppose that transactions with both suppliers $j, \bar{j}$ generate the same level of profit and have the same level of risk, in other words, $\Pi_{\bar{j}} = \Pi_j$ and $\varepsilon_{\bar{j}}\phi_{\bar{j}} = \varepsilon_j\phi_j$. This situation is shown in Figure 2.6.b. Here, the function $f(\psi_{jk})$ and the expected profit $\psi_{jk}\Pi_j + (1 - \psi_{jk})\Pi_{\bar{j}}$ become horizontal lines and are located above the level of β. In this case, the solution to problem (2) is indeterminate since all values $0 \leq \psi^*_{jk} \leq 1$ bring about the same level of profit. As a result, the buyer is indifferent towards engaging in a transaction with either supplier, since both offer the buyer the same level of profit and risk. If any of the two suppliers raises slightly the level of profit (or the risk associated with it drops slightly), then it will immediately supply the buyer and leave its competitor outside of the market. This outcome is similar to that of the Bertrand model, which converges to a solution where the price equals the marginal cost; which corresponds to the outcome under perfect competition as well.

2.3.1.3 Probabilistic choice with respect to price

The buyer's probabilistic choice can also be analyzed in relation to other variables that impact profits directly, like for example, the selling prices set by the suppliers. This analysis is essential for the understanding of price setting behavior in the network, which will be explained in further detail later in this section.

Let $p^{\mathfrak{m}}_{jk}$ and $p^{\mathfrak{m}}_{\bar{j}k}$ denote the prices set by the regular and the rival suppliers, respectively. Because these prices are included in the buyer's costs, we have:

$$\frac{\partial \Pi_j}{\partial p^{\mathfrak{m}}_{jk}} \leq 0\,, \qquad \frac{\partial \Pi_{\bar{j}}}{\partial p^{\mathfrak{m}}_{\bar{j}k}} \leq 0$$

and as a result:

$$\frac{\partial \psi^*_{jk}}{\partial p^{\mathfrak{m}}_{jk}} = \frac{\partial \psi^*_{jk}}{\partial \Pi_j}\frac{\partial \Pi_j}{\partial p^{\mathfrak{m}}_{jk}} \leq 0, \qquad \frac{\partial \psi^*_{jk}}{\partial p^{\mathfrak{m}}_{\bar{j}k}} = \frac{\partial \psi^*_{jk}}{\partial \Pi_{\bar{j}}}\frac{\partial \Pi_{\bar{j}}}{\partial p^{\mathfrak{m}}_{\bar{j}k}} \geq 0$$

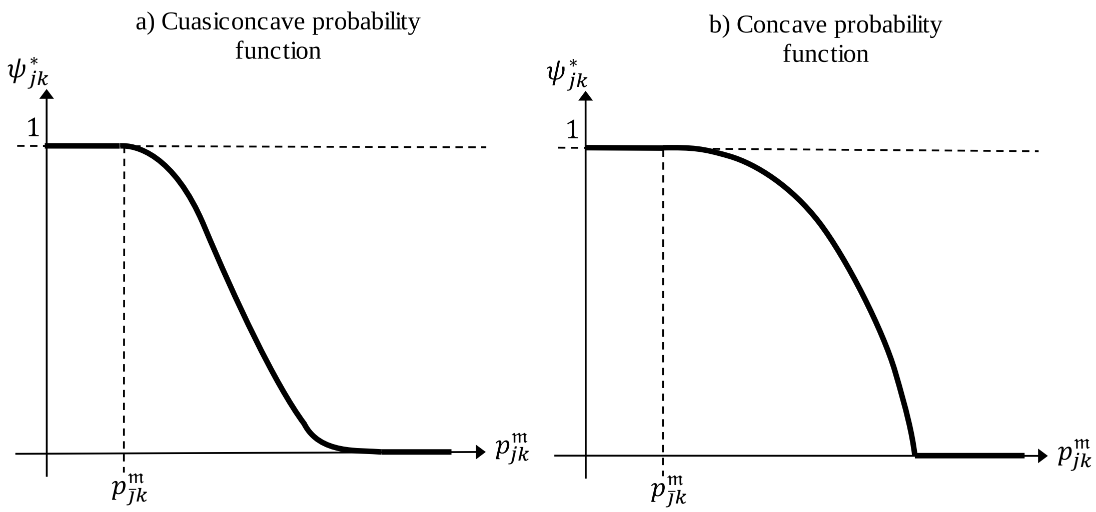
In other words, the probability ψ^*_{jk} increases as price $p^{\mathfrak{m}}_{jk}$ decreases or as price $p^{\mathfrak{m}}_{\bar{j}k}$ rises. This behavior is very intuitive given the rivalry that exists between both suppliers to sell their products.

Now, if we consider that the buyer demands its inputs from the regular supplier j when this supplier offers the lower price (in other words, the outcome is a monopoly that favors the regular supplier), then we have:

$$\psi_{jk}^{*} = 1 \Leftrightarrow \ p_{jk}^{\mathfrak{m}} \leq p_{\bar{j}k}^{\mathfrak{m}}$$

Based on these results, the probability ψ_{jk}^{*} may adopt the following functional forms:

Figure 2.7. Probability ψ_{jk}^{*} function



Note. Both figures illustrate the probability function's behavior that stems from the problem of probabilistic choice in relation to the price set by the regular supplier $p_{jk}^{\mathfrak{m}}$ and the price set by the rival supplier $p_{\bar{j}k}^{\mathfrak{m}}$. The abscissa's axis represents the price offered by the regular supplier $p_{jk}^{\mathfrak{m}}$ and the ordinate's axis shows the probability of transaction ψ_{jk}^{*}. Both are decreasing functions and equal 1 when $p_{jk}^{\mathfrak{m}} < p_{\bar{j}k}^{\mathfrak{m}}$.

As can be seen, they are decreasing functions that equal 1 when $p_{jk}^{\mathfrak{m}} < p_{\bar{j}k}^{\mathfrak{m}}$, and they can be quasiconcave or concave depending on the second order conditions assumed for f and g.[43]

[43] Given the conditions $f' > 0, g_1 > 0, g_2 > 0, g_{11} < 0, g_{22} < 0, g_{12} > 0$, it can be shown that with constant- returns-to-scale technologies the probability of transacting with the rival supplier $\bar{j}$ is concave with respect to the price $\left(\frac{\partial^2 \psi_{\bar{j}k}^{*}}{\partial p_{\bar{j}k}^{\mathfrak{m}\,2}} < 0\right)$. However, under these same circumstances, the probability of transacting with the regular supplier j is not necessarily concave $\left(\frac{\partial^2 \psi_{jk}^{*}}{\partial p_{jk}^{\mathfrak{m}\,2}} \lessgtr 0\right)$.

2.3.2 Agents' behavior in the model

2.3.2.1 Intermediate good firms

Each firm $j \epsilon J^{\mathfrak{m}}$ makes a decision which consists in finding the price vector $p_j^{\mathfrak{m}} = \left(p_{j1}^{\mathfrak{m}}, \dots, p_{jN}^{\mathfrak{m}}\right)' \in \mathbb{R}_+^N$ for selling good $a(j)$ to its potential buyers in the network, in a way that maximizes its expected profit over the set of realizations Ω.[44]

Formally, this problem can be formulated in the following way:

$$\begin{aligned} &\max_{p_j^{\mathfrak{m}}} E\left[\pi_j \mid p_j^{\mathfrak{m}}, p_{-j}^{\mathfrak{m}}\right] = \sum_{\omega \in \Omega} \Psi\left(\omega \mid p_j^{\mathfrak{m}}, p_{-j}^{\mathfrak{m}}\right) \ \pi_j\left(p_j^{\mathfrak{m}}, p_{-j}^{\mathfrak{m}}, \omega\right) \\ &p_{jk}^{\mathfrak{m}} \geq 0, \qquad \forall k \in \Delta_j^{+}(R) \\ &p_{jk}^{\mathfrak{m}} = 0, \qquad \forall k \notin \Delta_j^{+}(R) \end{aligned} \tag{4}$$

where $p_{-j}^{\mathfrak{m}} = \left(p_1^{\mathfrak{m}}, \dots, p_{j-1}^{\mathfrak{m}}, p_{j+1}^{\mathfrak{m}}, \dots, p_{N^{\mathfrak{m}}}^{\mathfrak{m}}\right) \in \mathbb{R}_+^N \times \mathbb{R}_+^{N^{\mathfrak{m}}-1}$ are the vectors of the prices set by the firms that produce intermediate goods, except for firm j; $\Delta_j^{+}(R) = \{k \epsilon V \mid (j,k) \in \Gamma\}$ is the set of potential buyers for the firm j; and $\Psi\left(\omega \mid p_j^{\mathfrak{m}}, p_{-j}^{\mathfrak{m}}\right)$ is the conditional probability of realization $\omega \in \Omega$ given prices $p_j^{\mathfrak{m}}$ and $p_{-j}^{\mathfrak{m}}$. Notice that $p^{\mathfrak{m}} = \left(p_j^{\mathfrak{m}}, p_{-j}^{\mathfrak{m}}\right) \in \mathbb{R}_+^N \times \mathbb{R}_+^{N^{\mathfrak{m}}}$.

This probability is defined by the following function:

$$\Psi(\omega \mid p^{\mathfrak{m}}) = \prod_{j \in J} \prod_{i \in b(k)} \prod_{j \in c(k,i)} \left(\psi_{jk}\right)^{\gamma_{jk}(\omega)}$$ [45]

where ψ_{jk} is the probability that buyer s demands the input from its regular supplier $r \in c(s,i)$; and $\gamma_{jk}(\omega)$ is the function that represents the inclusion of transaction (j,k) in the set of transactions $\hat{\Gamma}(\omega)$ in realization $\omega \in \Omega$.[46]

---

[44] In the literature, there are contributions that examine price setting in markets structured in a network, this is the case of Bimpikis, Ehsani, and Ilkiliç (2014) y Baqaee (2016). Unlike those studies, our model involves a price setting behavior that considers the buyers' probabilistic choice in the network.

[45] Recall that $\forall \omega \in \Omega, \forall k \in J, \forall i \in I \ \left|\overleftarrow{\Delta}_{ki}(\omega)\right| = 1$, which is why $\sum_{\omega \in \Omega} \Psi(\omega \mid p^{\mathfrak{m}}) = 1$ holds.

[46] $\gamma_{jk}(\omega) = 1 \iff (j,k) \in \hat{\Gamma}(\omega)$; $\gamma_{jk}(\omega) = 0 \iff (j,k) \notin \hat{\Gamma}(\omega)$

As was stated in the previous section, the probabilities ψ_{jk} are decreasing functions with respect to the regular supplier's price $p^{\mathfrak{m}}_{jk}$, and increasing functions with respect to the rival supplier's price $p^{\mathfrak{m}}_{\bar{j}k}$.

$$\forall k \in J, \forall i \in b(k), \qquad \psi_{jk} = \psi_{jk}\left(p^{\mathfrak{m}}_{jk}, p^{\mathfrak{m}}_{\bar{j}k}\right) \;\; j, \bar{j} \in c(k,i)$$

$$\frac{\partial \psi_{jk}}{\partial p^{\mathfrak{m}}_{jk}} \leq 0, \frac{\partial \psi_{jk}}{\partial p^{\mathfrak{m}}_{\bar{j}k}} \geq 0$$

Notice that due to complementarity, we have $\psi_{\bar{j}k} = 1 - \psi_{jk}$.

On the other hand, $\pi_j\left(p^{\mathfrak{m}}_j, p^{\mathfrak{m}}_{-j}, \omega\right)$ represents the profit obtained by firm j for transacting in realization $\omega \in \Omega$. This function is determined by:

$$\pi_j\left(p^{\mathfrak{m}}_j, p^{\mathfrak{m}}_{-j}, \omega\right) \; = \sum_{k \in \Delta^{+}_{j}(\omega)} p^{\mathfrak{m}}_{jk} \, z_{k,a(j)}\left(y_k, p^{\mathfrak{m}}_{jk}, p^{\mathfrak{m}}_{-j}, w(\omega)\right) \; - \; c_j\left(y_j, p^{\mathfrak{m}}_{-j}, w(\omega)\right)$$

Function $z_{k,a(j)}\left(y_k, p^{\mathfrak{m}}_{jk}, p^{\mathfrak{m}}_{-j}, w(\omega)\right)$ is the conditional demand of firm k with respect to good $a(j)$, which depends on the level of production y_k, price $p^{\mathfrak{m}}_{jk}$, input $a(j)$, prices $p^{\mathfrak{m}}_{-j}$ and the wage $w(\omega)$. On the other hand, function $c_j\left(y_j, p^{\mathfrak{m}}_{-j}, w(\omega)\right)$ is the minimum cost function for firm j, which is defined by total production $y_j = \sum_{k \in \Delta^{+}_{j}(\omega)} z_{k,a(j)}$, prices $p^{\mathfrak{m}}_{-j}$ and the wage $w(\omega)$. These functions for a CES type technology with constant returns to scale are shown in Appendix D. [47]

Notice that for both functions $z_{k,a(j)}\left(y_k, p^{\mathfrak{m}}_{jk}, p^{\mathfrak{m}}_{-j}, w(\omega)\right)$ and $c_j\left(y_j, p^{\mathfrak{m}}_{-j}, w(\omega)\right)$ the argument is the wage $w(\omega)$, which is subject to the realization of $\omega \in \Omega$. The reason behind this feature is simple. The labor market is assumed to be perfectly competitive, therefore the wage depends intrinsically on the behavior of the rest of economic agents and consequently, on the way in which these engage in transactions in each realization in the production network.

[47] Unlike in the case of a competitive market setting, in an oligopoly it is not necessary to assume decreasing returns to scale to guarantee the existence of a unique solution that maximizes firms' profits. To guarantee such a solution, it suffices to assume decreasing marginal returns, which can be achieved by using a constant-returns-to-scale technology and constant capital. We thank Carlos Uribe for this observation.

Assuming the second order effects of price setting are non-existent $\left(i.e. \frac{\partial y_k}{\partial p^{\mathrm{m}}_{jk}} = 0\ \forall j, k \mid j \neq k\right)$[48], the first order conditions of the problem (4) to reach an interior solution are:

$$\forall k \in \Delta_j^+(R), \sum_{\omega \in \Omega} \left[\frac{\partial \Psi}{\partial p^{\mathrm{m}}_{jk}} \pi_j(p^{\mathrm{m}}, \omega) + \Psi(\omega \mid p^{\mathrm{m}}) \frac{\partial \pi_j}{\partial p^{\mathrm{m}}_{jk}} \right] = 0$$

where:

$$\frac{\partial \Psi}{\partial p^{\mathrm{m}}_{jk}} = \Psi \frac{\partial \psi_{jk}}{\partial p^{\mathrm{m}}_{jk}} \left(\frac{\gamma_{jk}(\omega)}{\psi_{jk}} - \frac{1 - \gamma_{jk}(\omega)}{1 - \psi_{jk}} \right)$$

$$\frac{\partial \pi_j}{\partial p^{\mathrm{m}}_{jk}} = \gamma_{jk}(\omega) \left(z_{k,a(j)} + p^{\mathrm{m}}_{jk} \frac{\partial z_{k,a(j)}}{\partial p^{\mathrm{m}}_{jk}} - \frac{\partial c_j}{\partial y_j} \frac{\partial z_{k,a(j)}}{\partial p^{\mathrm{m}}_{jk}} \right)$$

For the sake of simplicity, this system of equations may be represented using the following reaction function for each firm $j \in J^{\mathrm{m}}$:

$$p^{\mathrm{m}}_j = F_j\left(p^{\mathrm{m}}_{-j}, W, Y_{-j}\right) \tag{5}$$

where $W \in \mathbb{R}_+^{|\Omega|}$ is a vector that contains the wages $w(\omega)$ corresponding to the realizations $\omega \in \Omega$; $Y_{-j} \in \mathbb{R}_+^{N-1} \times \mathbb{R}_+^{|\Omega|}$ is a matrix where the columns represent the production vectors $y_{-j}(\omega) = \left(y_1(\omega), \dots, y_{j-1}(\omega), y_{j+1}(\omega), \dots, y_N(\omega)\right)' \in \mathbb{R}_+^{N-1}$ of all firms except for firm j, for each realization $\omega \in \Omega$.

It is worth underlining that, as stated in formulation (4), firms' decisions may not be optimal *ex post*. This result arises from the probabilistic nature of the buyers' choice and the ability firms J^{m} possess to influence this choice through their prices setting behavior (which implies that $\Psi(\omega \mid p^{\mathrm{m}})$ is endogenous). In this sense, selling prices set by firms J^{m} will not necessarily maximize their expected profits after any realization $\omega \in \Omega$ takes place, they will only maximize their expected profits before this realization happens.[49]

[48] Under this assumption, a firm's behavior only takes into account the direct effect its prices have on its buyers' demand, and not the indirect effect on their output (or on the output of other firms in the network). The algorithm for the determination of the General Market Equilibrium suppresses this limitation through the iterative determination of both direct and indirect effects.

[49] If the firm can decide its selling prices after some realization (in order to maximize its ex-post profit), the best strategy for the firm would be to set a monopoly price. In this case, the firm would know with certainty

2.3.2.2 Final good firms

Firms J^{g} carry out their activities in perfectly competitive markets. These firms decide how much to produce in each realization $\omega \in \Omega$ seeking to maximize their expected profits and taking the prices of final consumption goods as given.

Formally, this problem can be formulated as follows, for firm $j \epsilon J^{\mathrm{g}}$:

$$\max_{y_j(\omega)} \sum_{\omega \in \Omega} \Psi(\omega \mid p^{\mathrm{m}}) \cdot \pi_j \left(y_j(\omega) \right) \tag{6}$$

where $y_j(\omega) \in \mathbb{R}_+$ is firm j's output in realization $\omega \in \Omega$; and $\pi_j \left(y_j(\omega) \right)$ is the profit function. This function is defined as:

$$\pi_j \left(y_j(\omega) \right) = p^{\mathrm{g}}_{a(j)}(\omega)\, y_j(\omega) - c_j \left(y_j(\omega), p^{\mathrm{m}}_{-j}, w(\omega) \right)$$

where $p^{\mathrm{g}}_{a(j)}(\omega) \in \mathbb{R}_+$ is the price of good $a(j) \in I^{\mathrm{g}}$ which is produced by firm in realization $\omega \in \Omega$, and $c_j \left(y_j(\omega), p^{\mathrm{m}}_{-j}, w(\omega) \right)$ is the minimum cost function for this realization. This function for the case of a CES type technology with constant returns to scale is presented in Appendix D. As can be observed in this formulation, price $p^{\mathrm{g}}_{a(j)}(\omega)$ is dependent on the realization of $\omega \in \Omega$, since its determination is carried out in perfectly competitive markets (just as is the case of the wage $w(\omega)$).

In solving this problem, we have that firm $j \epsilon J^{\mathrm{g}}$ decides to produce the quantity for which its marginal cost is equivalent to the market price.

$$\forall \omega \in \Omega, \qquad \frac{\partial c_j \left(y_j(\omega), p^{\mathrm{m}}_{-j}, w(\omega) \right)}{\partial y_j(\omega)} = p^{\mathrm{g}}_{a(j)}(\omega)$$

These conditions bring about the following supply functions $y_j\left(\omega, p^{\mathrm{m}}, p^{\mathrm{g}}(\omega), w(\omega)\right)$.

In contrast with firms J^{m}, firms J^{g} have no way of influencing the probabilistic choice of buyers in the network, which is why the probability $\Psi(\omega / p^{\mathrm{m}})$ associated with their transactions is exogeneous. In this sense, these firms' decision may be optimal both *ex*

---

the transactions of the productive network, without the risk of losing potential buyers due to the rivalry with other suppliers.

ante and *ex post*; in other words, its supply may be optimal even after any realization $\omega \in \Omega$ takes place.

2.3.2.3 The Representative household

The representative household maximizes its expected utility; for this purpose, it decides how much of each good it will demand and how much labor it will supply in each realization $\omega \in \Omega$, subject to a budget restriction. In this restriction, the household receives part of its income from supplying labor, but also from owning the firms' capital. Formally, this agent's behavior may be formulated as follows:

$$\begin{aligned} &\max_{x(\omega),l_h(\omega)} \sum_{\omega\in\Omega} \Psi(\omega \mid p^{\mathrm{m}}) \cdot u\big(x(\omega), l_h(\omega)\big) \\ &\sum_{i\in I^{\mathrm{g}}} p_i^{\mathrm{g}}(\omega) x_i(\omega) = \pi(\omega) + w(\omega)\cdot l_h(\omega), \qquad \forall\omega \in \Omega \end{aligned} \tag{7}$$

where $u(\, x(\omega), l_h(\omega)\,)$ is the consumer's utility function, $x(\omega) = \big(x_1(\omega), \dots, x_{M^{\mathrm{g}}}(\omega)\big)' \in \mathbb{R}_+^{M^{\mathrm{g}}}$ is the consumption vector for the realization of $\omega \in \Omega$, $p^{\mathrm{g}}(\omega) = \Big(p_1^{\mathrm{g}}(\omega), \dots, p_{M^{\mathrm{g}}}^{\mathrm{g}}(\omega)\Big)' \in \mathbb{R}_+^{M^{\mathrm{g}}}$ is the price vector for final consumption goods, $l_h(\omega) \in \mathbb{R}_+$ is the supply of labor and $\pi(\omega)$ is the firms' total profits.

$$\pi(\omega) = \sum_{j\epsilon J^{\mathrm{m}}} \pi_j(\omega) + \sum_{j\epsilon J^{\mathrm{g}}} \pi_j(\omega)$$

The solution to this problem generates Marshallian demand functions $x_i\big(\omega, p^{\mathrm{g}}(\omega), w(\omega), \pi(\omega)\big)$ and labor supply functions $l_h\big(\omega, p^{\mathrm{g}}(\omega), w(\omega), \pi(\omega)\big)$. The corresponding functions for CES type preferences are presented in Appendix D.

It is worth noticing that the conditional probability $\Psi(\omega \mid p^{\mathrm{m}})$ is an exogeneous variable in the representative household' decisions, same as was the case for the firms that produce final consumption goods. Hence, the representative household' decisions are optimal both *ex ante* and *ex post.*

2.3.3 General Market Equilibrium (GME)

The model's equilibrium, which we name the General Market Equilibrium (GME), is determined by the behavior and interactions of the firms and the representative household in the production network. This equilibrium denotes a system of prices and quantities which the firms and the household have decided upon by making the decisions that are in their best interests, in accordance with the formulation of the problems detailed previously.

It is worth noting that these decisions are optimal *ex ante*, as the economic agents maximize the expected value of their profits or utility. As has been shown, only for households and firms producing final consumption goods these decisions are also optimal *ex post*; while for intermediate good firms, not necessarily. On this matter, it is essential to establish certain considerations to ensure that the buying and selling transactions are executed in any realization $\omega \in \Omega$, even those where the intermediate good firms may obtain profits below those expected initially.

2.3.3.1 Assumptions for the equilibrium and definition of the GME

Assumption 1. Ex post local equilibirum in the intermediate goods market

The intermediate goods suppliers have the right to set the price of their products; while the buyers of these products have the right to establish the quantity they demand, without the suppliers refusing to produce the quantity the buyers require once prices have been set.

In formal terms, this assumption states the following[50]:

$$\forall \omega \in \Omega, \forall j \in J^{\mathrm{m}}, \qquad y_j(\omega) = \sum_{k \in \Delta_j^+(\omega)} z_{k,a(j)}\left(y_k, p_{jk}^{\mathrm{m}}, p_{-j}^{\mathrm{m}}, w(\omega)\right)$$

Assumption 2. Ex post global equilibrium in the final consumption goods market

Firms that produce final consumption goods and the representative household have the same right to determine their supply and demand of goods, respectively.

Formally, from this assumption we have:

$$\forall \omega \in \Omega, \forall i \in I^{\mathrm{g}}, \qquad x_i\left(\omega, p^{\mathrm{g}}(\omega), w(\omega), \pi(\omega)\right) = \sum_{j \in J^{\mathrm{g}} \mid a(j)=\{i\}} y_j\left(\omega, p^m, p^{\mathrm{g}}(\omega), w(\omega)\right)$$

[50] This assumption cancels the possibility of rationing and introduces a local equilibrium aspect; however, as has been stated previously, it does not ensure that the firms' decisions are optimal *ex post*.

Assumption 3. Ex post global equilibrium in the labor market

Firms and the representative household have the same right to determine their demand and labor supply, respectively

In formal terms, from this assumption we have:

$$\forall \omega \in \Omega, \qquad l_h\big(\omega, p^{\mathfrak{g}}(\omega), w(\omega), \pi(\omega)\big) = \sum_{i \in J} l_j\left(y_j, p^m, w(\omega)\right)$$

where l_j the demand for labor of firm $j \in J$.

For the sake of simplicity, these three assumptions will be represented using the following system of equations:

$$\forall \omega \in \Omega, \qquad H(x(\omega), y(\omega), p^{\mathfrak{g}}(\omega), w(\omega), p^{\mathfrak{m}}) = 0 \tag{8}$$

This system is characterized for being squared for prices $(p^{\mathfrak{g}}(\omega), w(\omega))$ and quantities $(x(\omega), y(\omega))$ in a realization $\omega \in \Omega$; taking as given prices $p^{\mathfrak{m}} \in \mathbb{R}_+^N \times \mathbb{R}_+^{N^{\mathfrak{m}}}$.

Based on these assumptions, the GME is defined as follows.

Definition 1. General Market Equilibrium (GME)

The General Market Equilibrium is the system of prices and quantities $\Lambda = (p^{\mathfrak{m}*}, \{x^*(\omega), y^*(\omega), p^{\mathfrak{g}*}(\omega), w^*(\omega)\}_{\omega \in \Omega})$ *which are determined by the agents' behavior as described in* (4), *and restricted by the ex post equilibrium conditions contained in* (8).

The GME shares some characteristics with the Walrasian equilibrium. For example, the prices of final consumption goods $p^{\mathfrak{g}*}(\omega)$ and the wage $w^*(\omega)$ are determined in such a way that, in each realization of $\omega \in \Omega$, all final goods markets and the labor market are cleared. In fact, due to the representative household's budget restriction, Walras' Law holds in system of equations (8), which is why it is enough to consider the local equilibrium in the intermediate goods market (assumption 1) and the global equilibrium in the final goods market (assumption 2) to attain the global equilibrium in the labor market (assumption 3). Despite sharing this similarity, the GME is not in itself a

Walrasian equilibrium, since it is treated as a phenomenon of price setting and price discrimination in the supply of intermediate goods.

Additionally, the GME has strong consonance with the concept of Nash equilibrium, since prices $p^{\mathfrak{m}*}$ are determined interdependently, taking into account each of the other firms' strategies, given their potential transactions in the production network. As stated in problem (4), each firm $j \epsilon J^{\mathfrak{m}}$ determines its price vector $p_j^{\mathfrak{m}*}$ in a way that maximizes its expected profits, assuming the rest of suppliers act in the same way in setting their prices $p_{-j}^{\mathfrak{m}*}$. It is this interaction the one that determines price setting in the network and the one that conditions the agents' behavior in the model.

As pointed out by Mas-Colell (1999), general equilibrium theory has a great potential if supplemented with game theory, since it can manage to break down the strategic simplicity and the anonymous interrelations assumptions which are present in competitive markets.[51] The concept defined in this work, the General Market Equilibrium (GME), constitutes a contribution in that respect, as it combines the idea of the Walrasian equilibrium with the Nash equilibrium, with the purpose of attaining a more reliable representation of the production network and its transactions.

The existence of this equilibrium with the following characteristics is demonstrated in Appendix E: a representative household with Cobb-Douglas preferences and no preference for leisure, firms with Marx-Leontief technologies in their demand for inputs, the production of each final consumption good is carried out by a representative firm and the trade-off function has a lineal specification.

---

[51] Mas Collel affirms that game theory has a close relationship with the theory of general equilibrium, to the point that it could be stated that the former is the latter's "cousin". He emphasizes that both theories manage the idea of equilibrium with agents' interaction, and that, in fact, both have supplemented each other methodologically. Nevertheless, according to the author, historically, both have been separated by the application of the principle of "Ockham's razor", since the most simplistic explanation that uses the assumption of price-taking firms has been preferred, instead of problematizing the economic phenomenon under the assumption of various agents making decisions in a strategic way.

2.3.3.2 Algorithm for finding the General Market Equilibrium

As can be observed in the previous section, the GME Λ is a Nash equilibrium within the structure of an *ex post* equilibrium.[52] In the literature, various techniques are employed for the numerical computation of a Nash equilibrium.

One of the simplest techniques to find this kind of equilibrium is to use the fixed-point method. This method uses reaction functions derived from the model to perform various iterations to find the equilibrium values. The present study uses this option and the algorithm that further explains its application is shown below:

Algorithm 1. Determination of the GME

1. Find all realizations in the production network $\omega \in \Omega$

2. Give an initial value to prices $p^{\mathfrak{m},0} \in \mathbb{R}_{+}^{N} \times \mathbb{R}_{+}^{N^{\mathfrak{m}}}$.

3. While $\|p^{\mathfrak{m},t+1} - p^{\mathfrak{m},t}\| \geq \epsilon$

3.1. Compute $p^{\mathfrak{m},t} = p^{\mathfrak{m},t-1}$

3.2. For every realization $\omega \in \Omega$

3.2.1. Find vectors $x(\omega), y(\omega), p^{\mathfrak{g}}(\omega), w(\omega)$ that solve for expression: $H(x(\omega), y(\omega), p^{\mathfrak{g}}(\omega), w(\omega), p^{\mathfrak{m},t}) = 0$.

3.2.2. Incorporate solution $y(\omega)$ in matrix Y_{-j}

3.3. For every firm $j \in J^{\mathfrak{m}}$,

3.3.1. Find the price vector $p_j^{\mathfrak{m},t+1} \in \mathbb{R}_{+}^{N}$ so that the following expression holds:

$p_j^{\mathfrak{m},t+1} = F_j\left(p_{-j}^{\mathfrak{m},t}, W, Y_{-j}\right)$.

EGM: $\Lambda \leftarrow p^{\mathfrak{m}*}$

4. For every realization $\omega \in \Omega$

4.1. Find vectors $x^*(\omega), y^*(\omega), p^{\mathfrak{g}*}(\omega), w^*(\omega)$ that are a solution to: $H(x^*(\omega), y^*(\omega), p^{\mathfrak{g}*}(\omega), w^*(\omega), p^{\mathfrak{m}*}) = 0$.

[52] von Heusinger (2009), and Facchinei and Kanzow (2010) carry out a thorough review of the algorithms used to determine computationally the Nash Equilibrium. The following methods stand out: decomposition methods, fixed-point methods that use homotopies and Nikaido-Isoda type functions and the conventional Newton methods to solve non-lineal systems of equations.

EGM: $\Lambda \leftarrow (p^{\mathrm{m}*}, \{x^*(\omega), y^*(\omega), p^{\mathrm{g}*}(\omega), w^*(\omega)\}_{\omega\in\Omega})$

This algorithm finds prices $p^{\mathrm{m},t} \in \mathbb{R}_+^N \times \mathbb{R}_+^{N^{\mathrm{m}}}$ iteratively so that they constitute solutions to the reaction functions (5) and solve for the local and global *ex post* equilibriums (8).

Essentially, the algorithm comprises four routines. Routine 1 generates the set of realizations in the production network Ω. These realizations satisfy the condition that states that all firms must offer their products to one or more buyers (or none); and demand their inputs from only one supplier. Routine 2 gives initial values $p^{\mathrm{m},0} \in \mathbb{R}_+^N \times \mathbb{R}_+^{N^{\mathrm{m}}}$ to the prices of intermediate goods. These values are generated randomly and are consistent with the structure of the production network R (which means $p_{jk}^{\mathrm{m},0} > 0$ if $(j,k) \in \Gamma$).

Routine 3 is composed of three subroutines. Subroutine 3.1 computes prices $p^{\mathrm{m},t} \in \mathbb{R}_+^N \times \mathbb{R}_+^{N^{\mathrm{m}}}$ for iteration t, in accordance with prices $p^{\mathrm{m},t-1} \in \mathbb{R}_+^N \times \mathbb{R}_+^{N^{\mathrm{m}}}$ which were determined in the previous iteration. Subroutine 3.2 solves the system of equations $H(x(\omega), y(\omega), p^{\mathrm{g}}(\omega), w(\omega), p^{\mathrm{m},t}) = 0$ for every realization $\omega \in \Omega$, taking prices $p^{\mathrm{m},t}$ as given. Then, the solution for the production vector $y(\omega)$ is incorporated in matrix Y_{-j}, as its ω-th column. Based on this result and on the prices estimated in previous iterations, subroutine 3.3 determines the price vector $p_j^{\mathrm{m},t+1} \in \mathbb{R}_+^N$ for firm $j \in J^{\mathrm{m}}$ by using the reaction function $p_j^{\mathrm{m},t+1} = F_j\big(p_{-j}^{\mathrm{m},t}, W, Y_{-j}\big)$. In solving both systems, H y F_j, the wage is taken as the numeraire in line with the satisfaction of Walras' Law $(\text{i.e.}\, W = (1,1,\dots,1);\ w(\omega) = 1)$. These subroutines are repeated until the difference between prices $p^{\mathrm{m},t}$ and $p^{\mathrm{m},t-1}$ is below the value of the error term ϵ.

Once the prices for intermediate goods $p^{\mathrm{m}*}$ have been determined, routine 4 repeats subroutine 3.2 until obtaining prices $p^{\mathrm{g}*}(\omega), w^*(\omega)$ and quantities $x^*(\omega), y^*(\omega)$ which define in a supplementary manner the GME Λ.

2.3.4 Firm bankruptcy

A crucial element in the analysis of the General Market Equilibrium is how the set of firms that participate in the production network is defined. In other words, the conditions that determine which firms remain active in their industries and which firms go bankrupt. In the present model, we assume that firms go bankrupt randomly and not in a deterministic manner as is usually assumed in the literature. This formulation is powerful, since it enables the representation of firm bankruptcy under unpredictable circumstances, especially given the economic system's high level of heterogeneity and complexity.

This approach demands certain considerations before analyzing it in conjunction with the notion of the GME. Let's assume the probability of firm bankruptcy is dependent on their level of profits. For example, the lower the level of profit, the higher the likelihood of firms exiting the industry and not engaging in transactions in the production network. In this sense, the GME may give off important signals about which firms would go bankrupt in the network, since firms' profits can be known from the prices and quantities exchanged in this equilibrium. However, to gain access to these signals, first it is necessary to know the set of firms that would previously make up the network and which would therefore condition the transactions that would take place in it. In other words, the GME will not be determined before identifying which firms are more likely to remain in the industry. This feature generates a sort of dynamic circularity between the GME's determination and firm exit, which must be incorporated in the model through the formulation of a stochastic process.

To this end, let's consider the process of firm permanence in the industry. Let $\{s_t\}_{t\geq 0}$ be a stochastic process that shows the set of active firms s_t in period t, so that $s_{t+1} \subseteq s_t$. This process's probability distribution is given by:

$$\begin{gathered} H(v_{t+1}, v_t, \dots, v_{t-r}) = \Pr(s_{t+1} = v_{t+1}/s_t = v_t, \dots, s_{t-r} = v_{t-r}) \\ \forall t \geq 0, \forall v_{t+1}, v_t, \dots, v_{t-r} \in \wp(J) \end{gathered} \tag{9}$$

where $\wp(J)$ denotes the set of subsets of J, which satisfy condition (1) and which have at least one supplier for the inputs demanded by each active firm in the network, in other words:

$$\forall s \in \wp(J), \forall k \in s, \forall i \in b(j), c(k,i) \neq \emptyset$$ [53]

[53] If an active firm $k \in s_t$ has only one supplier with respect to the input it requires (i.e. $|c(k,i)| = 1$), it will have monopoly power, regardless of whether it was previously a regular supplier or rival.

On the other hand, $H(v_{t+1}, v_t, \dots, v_{t-r})$ is the probability that firms v_{t+1} are active during period $t+1$, given that firms $v_t, \dots, v_{t-r}$, were active during the r previous periods.

Each set of firms s_t in the process $\{s_t\}_{t\geq 0}$ finds itself associated with a general market equilibrium Λ^{s_t} and with its corresponding space of realizations Ω^{s_t} in the production network.

It is assumed that firms go bankrupt independently, therefore we have the probability function $H(v_{t+1}, v_t, \dots, v_{t-r})$:

$$H(v_{t+1}, v_t, \dots, v_{t-r}) = \prod_{j\in v_{t+1}} \eta_j\left(\hat{\pi}_j \mid \Lambda^{v_t}, \dots, \Lambda^{v_{t-r}}\right)$$

where $\eta_j\left(\hat{\pi}_j \mid \Lambda^{v_t}, \dots, \Lambda^{v_{t-r}}\right)$ is the probability that firm j is active during period $t+1$, given the equilibria $\Lambda^{v_t}, \dots, \Lambda^{v_{t-r}}$ in r previous periods; and $\hat{\pi}_j = \left(\hat{\pi}_{j,t}, \dots, \hat{\pi}_{j,t-r}\right)' \in \mathbb{R}_+^r$ is the vector of observed profits for firm j in the last r periods. It must be emphasized that each value $\hat{\pi}_{j,t-k}, 0 \leq k \leq r$ in this vector denotes the profit obtained by firm j in realization $\omega \in \Omega^{v_{t-k}}$ which belongs to equilibrium $\Lambda^{v_{t-k}}$.

For the sake of simplicity, we will assume that the probability of firm j remaining in the network depends on the number of times it obtained a profit above zero in the last r periods. The larger this number, the higher its probability of remaining in the network. To do so, we consider the following function:

$$\eta_j\left(\hat{\pi}_j \mid \Lambda^{v_t}, \dots, \Lambda^{v_{t-r}}\right) = \left(\frac{1}{r}\sum_{k=1}^{r} \mathbf{1}\left(\hat{\pi}_{j,t-k} > 0\right)\right)^{\theta_j}, \qquad \theta_j < 1$$

where $\mathbf{1}\left(\hat{\pi}_{j,t-r} > 0\right)$ is the indicator function and θ_j is a bankruptcy sensitivity parameter.[54] Since firms producing final consumption goods operate in perfectly competitive markets, their profits are always positive in any realization of the network, and therefore, these firms will always trade in the market (i.e. $\eta_j = 1\ \forall j \in J^{g}$).

---

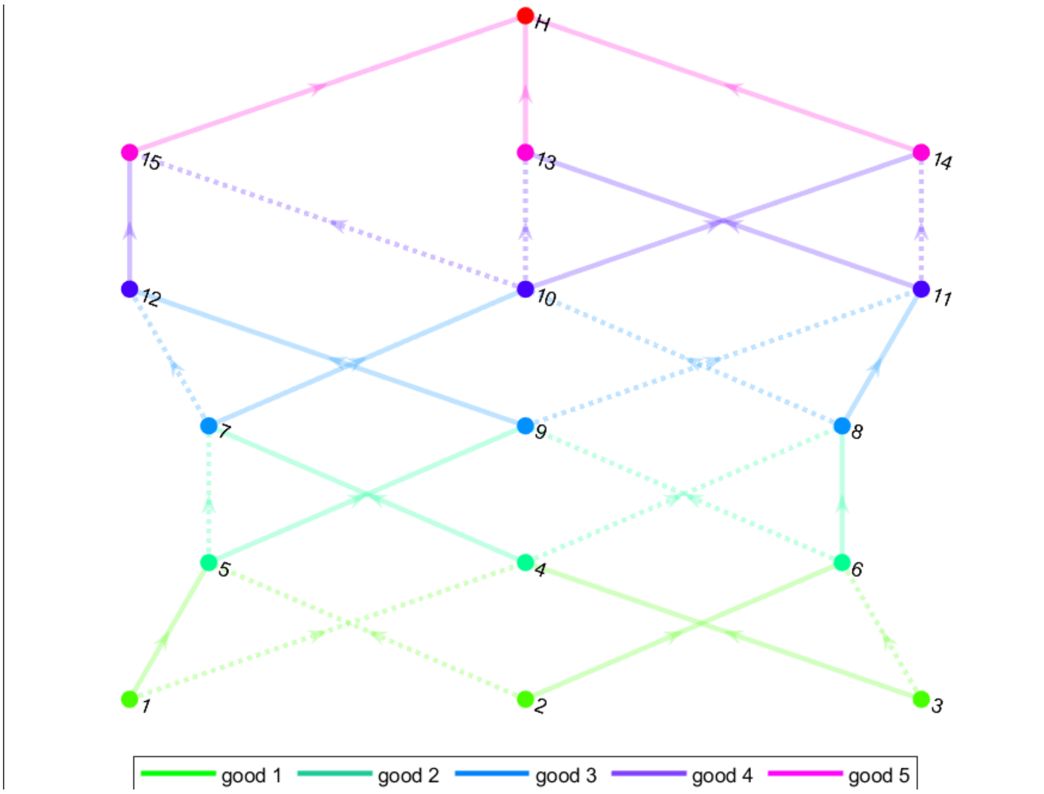
[54] The higher this constant, the more concave will the probability function be, and with it, the higher the firm's probabilities of remaining in the industry when the number of periods with positive profits is large.

2.4 Empirical analysis of the General Market Equilibrium

The purpose of this section is to show some of the GME's characteristics, to observe how firm productivity shocks may impact the equilibrium's determination and firm bankruptcy and analyze the propagation of these shocks in the network. It must be emphasized that this analysis is merely empirical. It is carried out on the basis of numeric simulations on a non-real production network, which satisfy the assumptions and rules of the model proposed in the previous section.

To do so, we consider a production network with 15 firms $J = \{j_1, j_2, \dots, j_{15}\}$, 5 goods $I = \{i_1, i_2, \dots, i_5\}$ and a representative household H, as shown in Figure 2.8. For illustrative purposes we only display transactions in the goods market.

Figure 2.8. Production Network used in the model's simulation



Note. This figure shows the production network over which simulations are run in the model. Each node denotes a firm, each edge represents a buying-selling transaction and each color identifies a good. If the edges' line is continuous, then transactions are carried out with the regular supplier; on the contrary, if the edges' line is dotted, then transactions occur with the rival supplier. For illustrative purposes, transactions in the labor market are omitted from the figure.

All transactions in the network are denoted by directed edges. As you may recall from the explanation in Figure 2.1, the color identifies the good that is being exchanged, for example, firm j_9 produces good i_3 (colored blue), which is acquired by firms j_{11}, j_{12}, and demands good i_2 (colored cyan) which is sold by firms j_5, j_6. On the other hand, the nature of the line distinguishes between the type of supplier: a continuous line represents a regular supplier, while a dotted line denotes a rival supplier. This distinction is not applied to the representative household, since its suppliers remain unchanged. For example, as a seller, firm j_9 is the regular supplier of firm j_{12} (continuous line) and the rival supplier of firm j_{11} (dotted line); whereas, as a buyer, firm j_9's regular supplier is firm j_5 (continuous line) and its rival supplier is firm j_6 (dotted line).

This network is built satisfying the assumptions that were established in the economic model, in other words: each market has at least one buyer and one supplier, no firm produces intermediate and final consumption goods simultaneously; all firms have two suppliers for every input they demand. To simplify the network's structure, it is assumed that firms are divided uniformly by levels of linkages, through a multipartite graph without cycles.[55] Additionally, it is assumed that each firms demands an intermediate good and labor (except for firms that are located on the first linkage level, and which demand only labor).

Regarding the economic agents' behavior, it is assumed that all firms use CES technologies with constant returns to scale and elasticities of substitution above 1. This technology's coefficients and parameters are generated randomly and are similar for firms located on the same linkage level. On the other hand, the representative household adopts CES preferences with an elasticity of substitution above 1. Technologies and preferences with elasticities of substitution greater than 1 ensure that the algorithm's performance in the determination of the GME is good.[56]

[55] In graph theory, a multipartite graph is a graph whose nodes may be divided in independent sets (meaning, a set where nodes are not adjacent to each other). For the purposes of the present work, each partition in this graph represent a linkage level in the network, which is conformed by firms that provide the same good and demand the same set of inputs. The firms' layout throughout the network ensures that each linkage level supplies only one different linkage, from firms that demand labor to those that produce final consumption goods.

[56] Firms that produce intermediate goods set their prices by observing their buyers' demand, which is why there is a risk that prices vary sharply when the price-elasticity of demand is low. This situation arises specially when the buyers' elasticity of substitution is close to zero (that is, when there exists a high degree of complementary between different inputs).

Finally, it is assumed that the market for intermediate goods has a pseudo-monopolistic structure. Here, the buyers' probabilistic choice is denoted by a 3rd order spline. In this spline, it is assumed that regular suppliers have a decreasing probability, which is equal to 1 when their price is below the rival supplier's, and which is equal to 0 when their price is above the rival supplier's.

Next, the GME is estimated in the initial period and a simulation of firm bankruptcy in the absence of productive shocks is carried out. This scenario constitutes our base scenario. Then, a 20% productivity shock is introduced in each firm's technology and its main effects are examined. These scenarios constitute the alternative scenarios (one scenario for each firm). In both scenarios, firm bankruptcy is simulated with 1,000 random trials in a time horizon of 30 periods. In each period, the probability of a firm remaining in the industry depends on the number of times it attained non-negative profits considering up to 5 lags ($r = 5$).

Numerical programming of the GME and firm bankruptcy was carried out in MATLAB.[57]

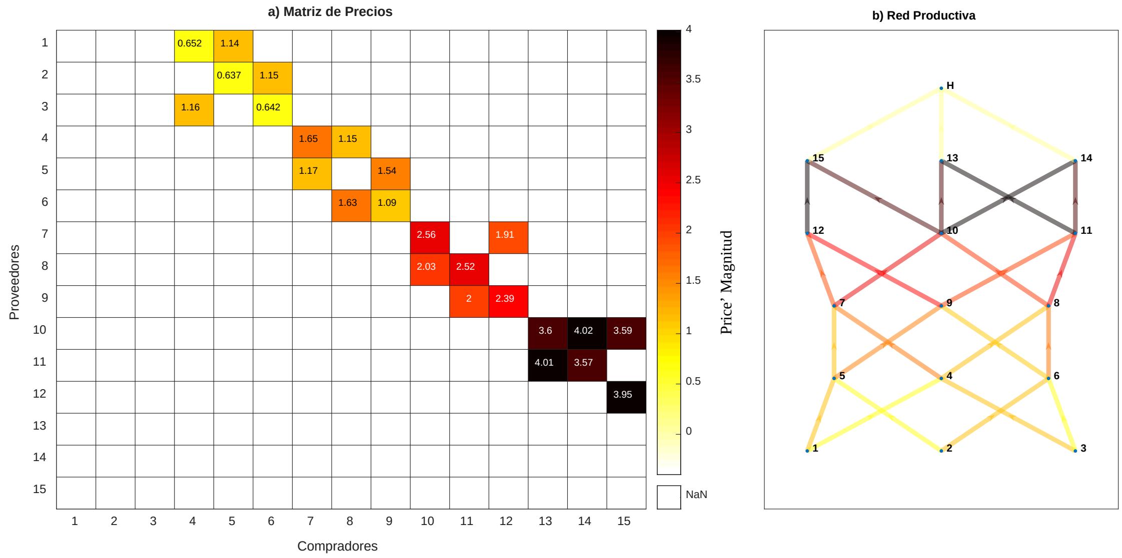
2.4.1 Base scenario

The production network in Figure 2.8 comprises $|\Omega| = 2^{12} = 4{,}096$ possible realizations. Each realization represents a different set of buying-selling transactions between firms. Recall that these realizations satisfy the restriction that states that firms must demand inputs from one supplier only and that they may offer their output to one or more buyers (or none) in the network.

Based on this set of realizations and taking into account the conditions for the production network's *ex post* equilibrium and the price reaction functions of each firm, the GME is determined for the initial period. The selling prices for intermediate goods $p^{\mathrm{m}*}$ that correspond to this equilibrium are shown below.

[57] The determination of the GME uses algorithms for optimization and non-linear systems of equations, while the modelling of firm bankruptcy uses stochastic simulation algorithms. The model's programming was carried out in supercomputer from the National Research and Education Network for Ecuador (CEDIA) and the supercomputer from Center for Mathematical Modelling in Key Areas for Development (MODEMAT). Its compilation requires around 4 to 5 days.

Figure 2.9. General Market Equilibrium for the base scenario (no shock). Intermediate goods prices. Initial period.



Note. Table a) shows the selling prices set by the suppliers for their potential buyers. The suppliers are arranged by rows, while the buyers are arranged by columns. The yellow-red gradation represents the magnitude of the price; the higher the intensity of the red coloration, the higher the offered price. Figure b) translates this color-coding scheme to the edges of the production network.

Each cell in matrix 2.9.a displays the selling price that the supplier (specified by row) sets for its potential buyer (specified by column) using a yellow-red gradation. The intensity of the red coloration is stronger, the higher the price. For example, cell (6, 9) shows that supplier j_6 sets its price at 1.09 monetary units for buyer j_9. From the cell's orange coloration, we may infer that this is a medium price level, compared to the rest of prices in the matrix. This coloration may also be observed on the edges of the network in Figure 2.9.b.

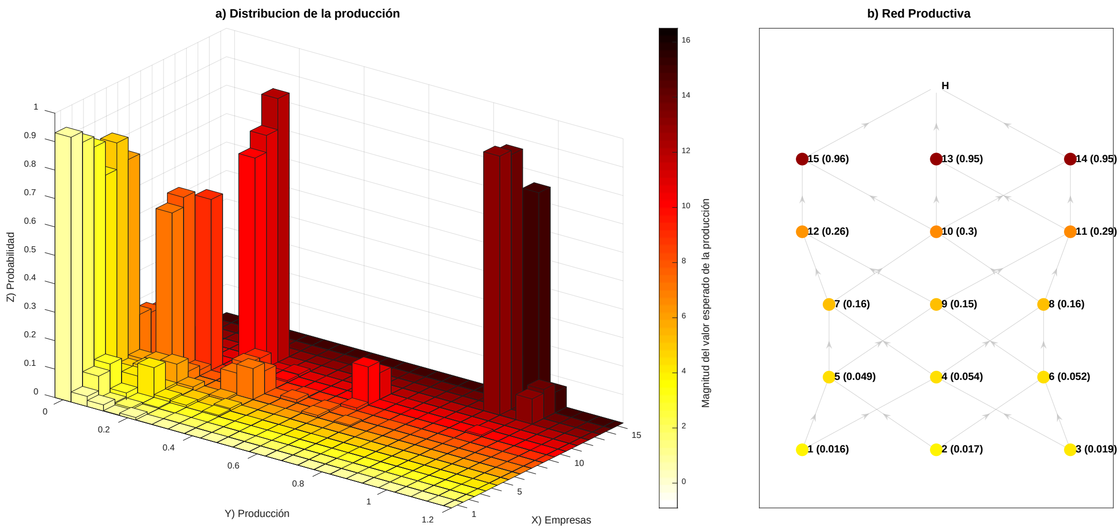
A quick inspection of the price matrix 2.9.a makes it possible to establish two characteristics of price determination in the GME. First, the price offered by the regular suppliers is always higher than that offered by the rival suppliers, for every buyer in the production network. This inference arises from observing the color variation in the cells belonging to the same column in matrix 2.9.a (alternatively, from the color variation observed in the edges that meet at the same node in the network in Figure 2.9.b. For example, firm j_9's regular supplier j_5 offers a selling price of 1.54 monetary units (colored orange), while the rival supplier j_6 offers a price equal to 1.09 (colored yellow-orange). As may be recalled, regular suppliers have the advantage of frequently engaging in transactions with their buyers and therefore, presenting a lower risk for them. This characteristic allows them greater power in setting their prices and therefore, a greater ability to fix selling prices above those fixed by the rival suppliers.

Second, prices rise along the productive linkages. This characteristic can be inferred by observing the increase in the red coloring's intensity of the cells in matrix 2.9.a, when moving from the upper left corner to the lower right corner (this characteristic is more clearly observed in the network in Figure 2.9.b, where the red coloring's intensity of the edges increases as we move from the first levels of the linkages, to the last). For example, supplier j_1, which is located on the first level of linkages, offers a vector price equal to [0.652 ; 1.14] with low level price values (colored yellow and light orange), while supplier j_{10}, which is located in the next-to-last level in the linkages, offers a vector price equal to [3.6 ; 4.02 ; 3.59] with high level values (colored carmine and dark red). The occurrence of this phenomenon is reasonable since firms must set selling prices which are above their marginal costs to maximize their profits. These prices intervene in the inputs demanded by their buyers, in turn, the buyers of the latter, and so on, as all firms exhibit the same price setting behavior. Consequently, prices must increase from transaction to transaction in the production network.

Another element that is characteristic of the GME is the production of firms $\{y(\omega)\}_{\omega\in\Omega}$. Recall that selling prices of intermediate goods in the GME condition the behavior of various variables in the economic system, including household consumption, final goods prices, labor demand and supply and even production. Next, the distribution of the output of the 15 firms introduced previously is shown, with their corresponding expected values.[58]

[58] Appendix D shows the distribution of the representative household's consumption and the prices of final consumption goods. Moreover, the most probable realizations of the production network are shown, with their respective probabilities.

Figure 2.10. General Market Equilibrium in the base scenario (no shock).
Firm Output. Initial Period.



Note. Figure a) is a multidimensional histogram of firms output, based on the empirical distribution function of the GME. The x-axis denotes the network's firms, the y-axis represents the logarithm of output and the z-axis denotes the probability. The yellow-red gradation indicates the expected value of production. The higher the color red's intensity is, the higher the expected value. Figure b) reproduces this gradation over the nodes in the network. The values in parenthesis show the expected output value of each node.

As can be observed, firms output rises when moving across the productive linkages in an ascending way. This remark can be observed in histogram 2.10.a, in the increasing intensity of the color red along the x-axis coordinates. It can also be visualized in the network in Figure 2.10.b, in the increasing intensity of the nodes' red color when moving from the first levels of linkages to the last. For example, if we analyze the network from one end to the other, we find that firm j_1 (which is located at the first linkage level) has an expected value below 0.0164 (colored light yellow), while firm j_{15} (which is located at the last linkage level) has a high expected value of 0.9591 (colored dark red).

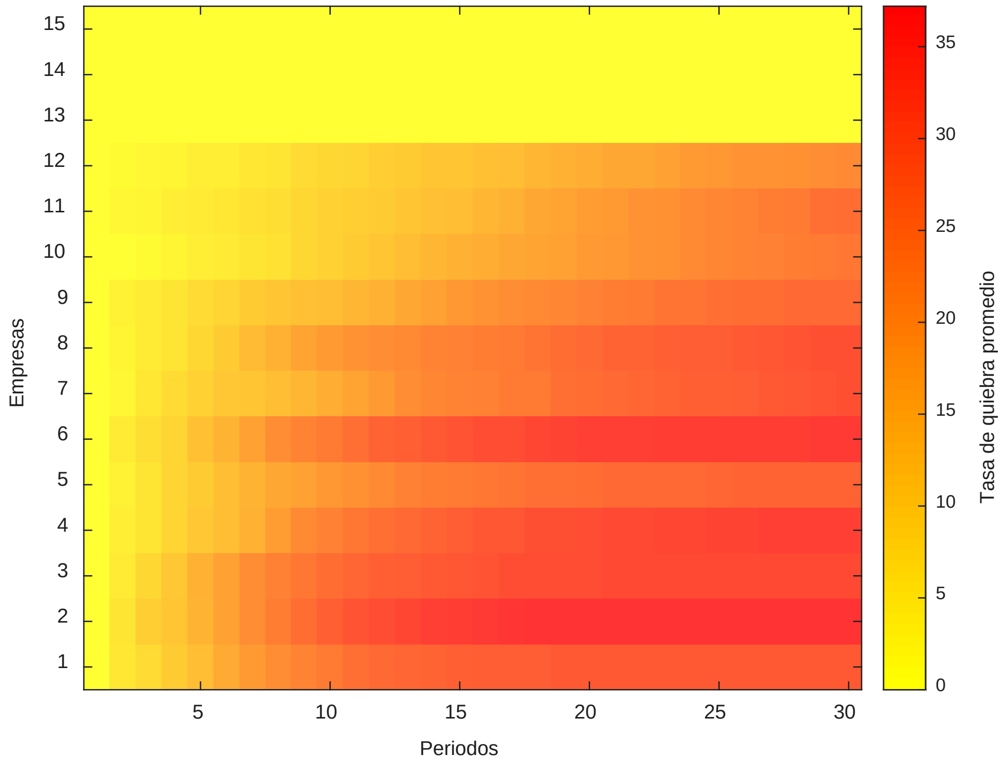
Two reasons underlie this increase in the expected value of output. First, there exists a multiplying effect on production caused by the structure of the production network. Each linkage level demands all inputs produced by the linkage found immediately before it, and labor, which is why its output must necessarily be larger. This effect extends from those firms which only demand labor (first linkage level) to those firms that satisfy the representative household's demand (final linkage level). Second, intermediate good firms and final good firms operate under different market structures.

The firms that produce intermediate goods work in pseudo-monopolistic markets, which is why in some occasions they may not produce any output. This may occur specially in realizations in the network in which the buyers decide not to engage in transactions with their suppliers because their offered prices are too high. These realizations cause a left bias on the distribution of output and therefore, reduce its expected value. On the contrary, firms that produce final consumption goods always produce a quantity above zero because they operate in competitive markets. Here, each firm produces at a level where its marginal cost equals the market price. Consequently, the output distribution is symmetric, and its expected value is higher.

The GME shown in Figures 2.9 and 2.10 represents the economic system's situation in the initial period. From this point on, the simulation of firm bankruptcy for 1,000 random trials is carried out, for a time horizon of 30 periods. The network's behavior in each period is determined by a new GME based on the firms that remained within the market up until that period. It is worth emphasizing that in this exercise, the permanence of each firm in the market is determined by the number of times the firm obtained a non-zero profit in the last 5 periods.

First, let's analyze the situation of each firm. Figure 2.11 illustrates firm bankruptcy in the production network throughout time by using a heat map. The rows on the map represent the firms, while the columns denote the time periods. The higher the intensity of the red color in each cell is, the higher the firm's average rate of bankruptcy in a specific time period.

Figure 2.11. Firm bankruptcy in the base scenario (with no shock).
Average Rate of Bankruptcy, by firm. 30 time periods.



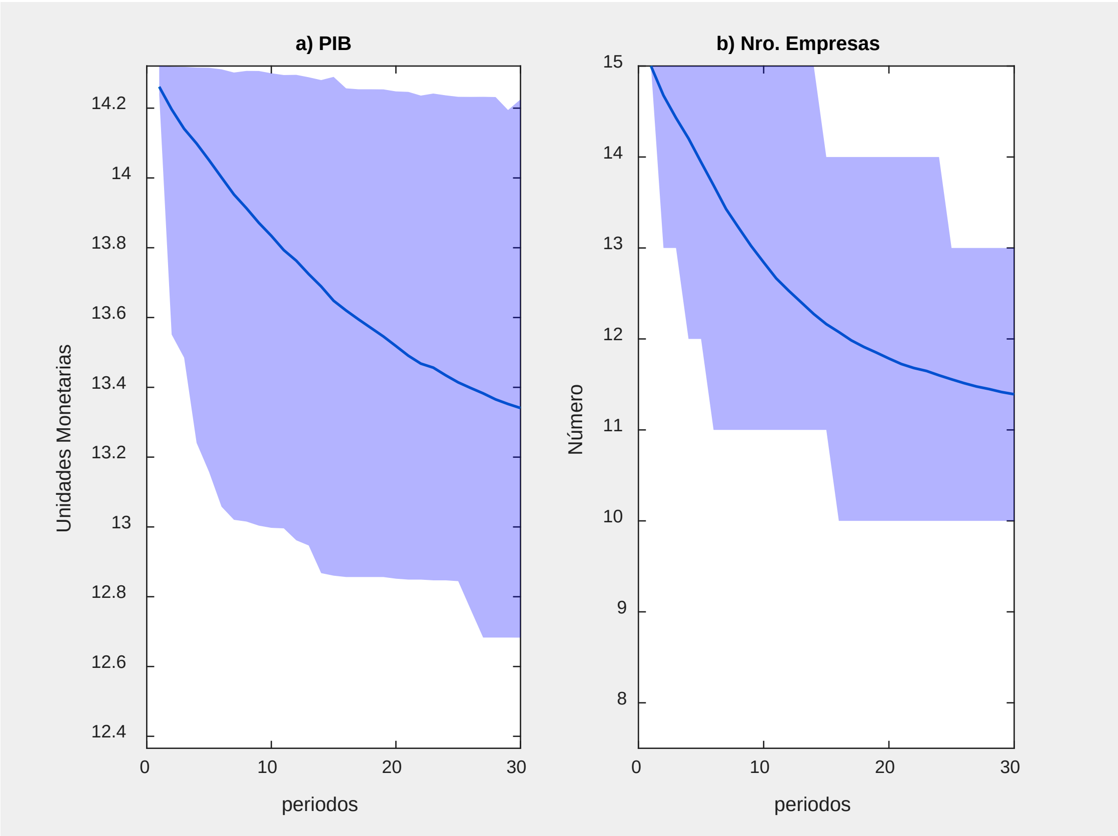
Note. This figure shows firms average rate of bankruptcy over a set of 1,000 random trials in a time horizon of 30 periods. Each row corresponds to a specific firm and each column corresponds to a specific time period. The yellow-red gradation specifies the magnitude of the average rate of bankruptcy. The higher the intensity of the color red is, the higher the firm's average rate of bankruptcy in the specific time period.

As observed in this figure, firms in the first linkage levels have a higher likelihood of going bankrupt and they do so faster; on the contrary, firms located at the last linkage levels are less likely to go bankrupt and they do so slower.

This observation excludes firms that produce final consumption goods, which never go bankrupt because they always exhibit positive profits. For example, firms j_2, j_3, j_4 (linkage levels 1 and 2) exit the industry starting at approximately period 5 and attain a maximum bankruptcy rate of 35% at the end of 30 periods. On the other hand, firms j_{10}, j_{11}, j_{12} (linkage level number 4) exit the industry starting at approximately period 17 and attain a maximum bankruptcy rate of 20%.

The high bankruptcy rate observed in the first levels of productive linkages can be explained by the way in which the network's structure conditions the activity of the firms. If a firm goes bankrupt, its suppliers have less chances of earning profits, which is why their probabilities of going bankrupt increase. This effect is transmitted from supplier to supplier, all the way down to the firms located at the first levels of linkages, where the propensity of exiting the industry is multiplied. It is necessary to underline that this effect is not transmitted to buyers, since there always exist another firm that can replace the firm that went bankrupt and which can provide the inputs required in the network. In aggregate terms, the economic system's total income and the number of firms that participate in transactions in the production network is shown in Figure 2.12. The thick line in each subfigure represents average GDP and the average number of firms in each time period, while the blue bands represent the confidence intervals, calculated at the 95% level.

Figure 2.12. Firm bankruptcy in the base scenario (with no shock).
GDP and total number of continuing firms. 30 time periods.



Note. Figures a) and b) show GDP and the total number of firms, respectively, over a set of 1,000 random trials in a time horizon of 30 periods. The thick line represents the general trend, while the blue bands represent the 95% confidence intervals.

As can be observed in Figure 2.12.b, firm bankruptcy reduces the set of firms that participate in the production network from 15 to approximately 11. According to Figure

12, firms with the higher likelihood of going bankrupt in this transition are firms j_2, j_3, j_4, j_6. This phenomenon reduces GDP approximately from 14.2 to 13.4 monetary units (Figure 2.12.a).

It is worth noticing that the reduction in GDP and the number of firms is not stabilized throughout time (meaning, there is no stationary state), because there is no mechanism in the model to create capital that may increase the number of firms in the production network.[59] Consequently, the reduction in GDP and the number of firms may continue if the number of time periods in the simulation is increased.

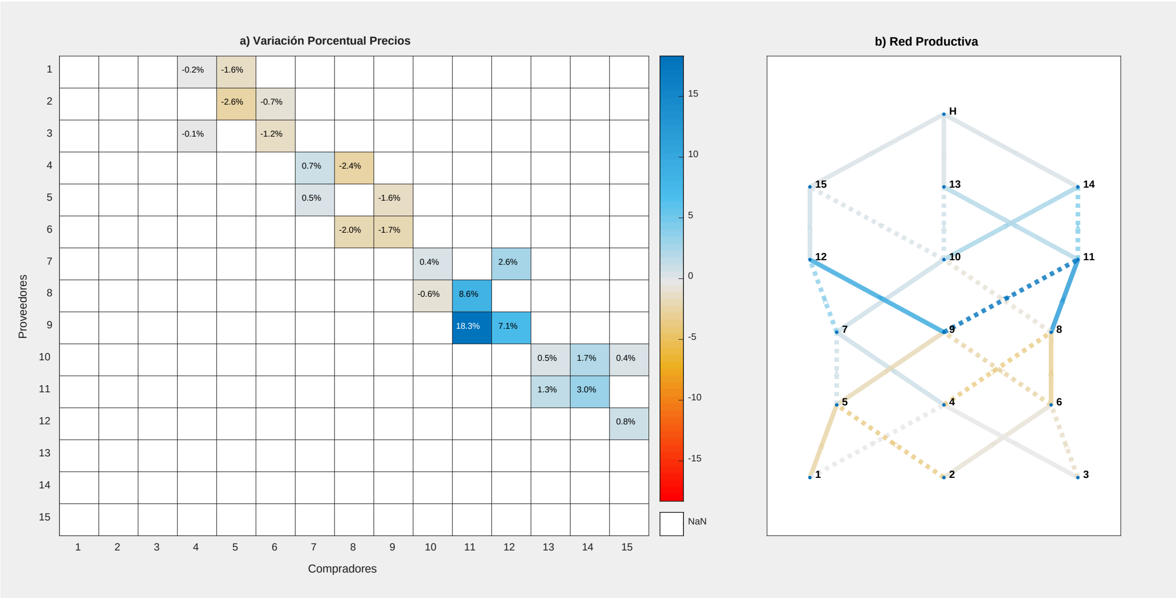
2.4.2 Alternative scenarios. Productivity shocks

This section analyzes how the GME and firm bankruptcy change when there are productivity shocks of 20% on firms' technologies. These shocks are simulated in an independent and separate way for each firm and represent what we had defined in the beginning as alternative scenarios. To understand the impact of these shocks and their propagation in the production network, we will center our attention in one single scenario; in this case, on the case of a productivity shock that affects firm j_9 because of its intermediate location in the production network. The rest of scenarios are shown in Appendix G.

First, we will examine how the GME changes in the initial period. As observed in table 2.13.a, a productivity shock on firm j_9 generates changes not only on its selling prices but also on the selling prices of the rest of firms in the network. Each cell in this table shows the percentage variation in the price set by the supplier (specified by row) for its potential buyer (specified by the column) using a red-blue gradation. This color coding indicates the sign and magnitude in the price variations. If the cell is red, then prices decrease; on the contrary, if the cell is blue, then prices increase. Each color's intensity depends on the magnitude of the absolute value's variation. This same color grading is applied to the edges in the network in Figure 2.13.b.

[59] To ensure a stationary state in the model, it may be assumed that there exists an amount of capital in the economy which may be invested in the creation of new firms. These firms must have similar (or improved) characteristics in relation to the firms that went bankrupt, so they may replace them and occupy their position in the network. This investment may depend on the profit a firm would expect and competition in the industry. Its incorporation may be implemented using a stochastic process, same as it was carried out for the inclusion of firm bankruptcy in the model.

Figure 2.13. GME in the alternative scenario. Productivity Shock of 20% on firm 9.
Intermediate Good Price variations. Initial period.



Note. Table a) shows the percentage variation in the selling prices set by the suppliers for their potential buyers. Suppliers are arranged by rows, while buyers are arranged by columns. The red-blue gradation denotes the sign and magnitude of the price variation. If red, then prices decrease; the higher the color red's intensity, the larger the price reduction. On the contrary, if blue, then prices rise; the higher the color blue's intensity, the larger the increase in prices. Figure b) replicates this color grading on the edges of the production network.

First of all, we observe that the productivity shock on firm j_9 increases its selling prices by 18.3% and 7.1% for buyers j_{11} and j_{12}, respectively. This increase is due to the increase in the marginal cost that the firm experiences after the productivity shock; this effect is stressed in those transactions where the firm participates as a rival supplier (since in these cases, the chances of losing a buyer because of a decision to increase the price are lower). This shock is propagated in the network generating two prevalent effects on the selling prices of the rest of the firms in the network: a horizontal effect and a cascade downstream effect.

The horizontal effect is an increase in the prices of firms that compete with the firm impacted by the shock. This effect can by distinguished by the blue color of the cells in rows 7, 8 and 9 in table 2.13.a and they are located in columns 11 and 12. It may also be visualized in the production network in Figure 2.13.b. in the blue color in the edges that stem from nodes 7, 8, and 9, and end in the same nodes 11 and 12. For example, we observe that competition between firms j_8 and j_9 for buyer j_{11} spreads the shock received by firm j_9 to firm j_8, increasing its selling price by 8.6% for buyer j_{11}. On the other hand, the downstream cascade effect represents the increase in prices that occurs from buyer to buyer in the network. This effect can be visualized in the blue color of the cells located below row 9 in table 2.13.a. Similarly, this effect may be observed in the blue colored edges above node 9 in the network shown in Figure 2.13.b. For example, we observe that the shock received by firm j_9 is diffused to its buyer j_{11}, which raises its prices by 1.3% and 3% for buyers j_{13} and j_{14}, respectively.

The explanation underlying both effects is simple. On the one hand, the horizontal effect is due to the interaction between firms when setting prices. If a firm increases its selling prices after a shock, then its buyers will start transacting with other competing suppliers. These suppliers, in view of the increase in demand, will also raise their selling prices, but will do so in a proportion lower than that corresponding to the impacted firm, because otherwise, they would lose the buyers that were secured initially. On the other hand, the downstream cascade effect can be explained by a cost factor. When a firm is affected by a negative shock, its marginal cost increases, forcing it to raise its selling prices. This effect, supplemented by the horizontal effect, increases costs for its buyers' who will also increase their selling prices to adjust to the cost increase. This effect is diffused from buyer to buyer in the network and diminishes as buyers are located farther from the impacted firm.

In Appendix G.1. we observe that the horizontal effect and the downstream cascade effect in the increase in prices is a characteristic of all alternative scenarios. There are scenarios like the one shown in Figure 2.13, where a decrease in upstream prices, in other words, from supplier to supplier. However, this phenomenon is not present in an absolute way for all prices and neither in a generalized way in all scenarios.[60]

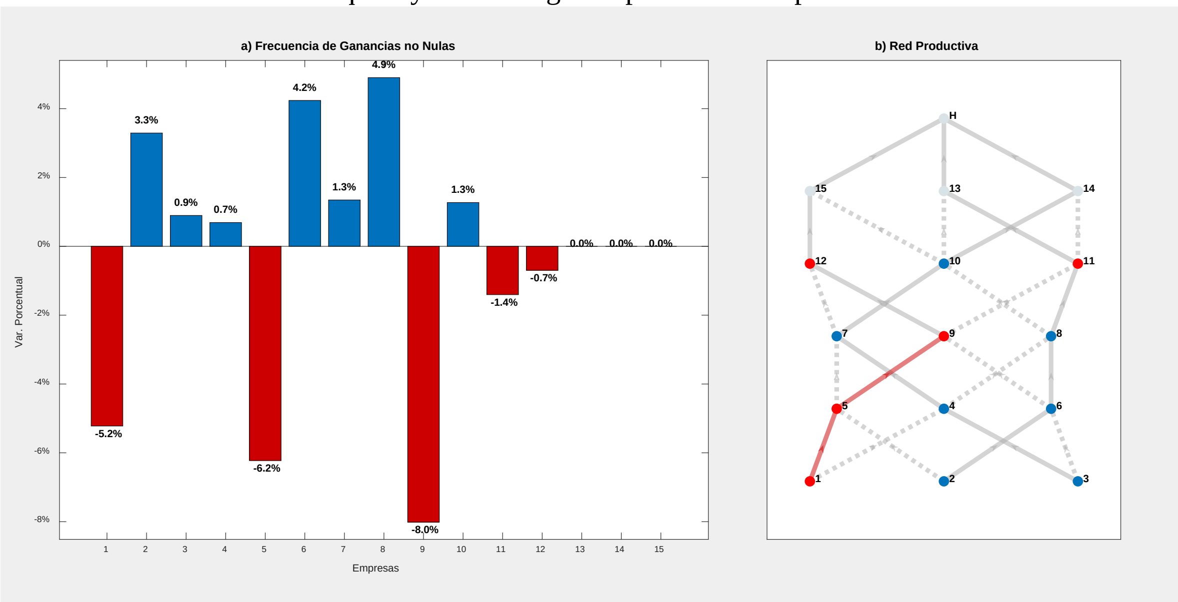
The increase in the prices of the firm that was impacted by the shock brings about three important implications in the analysis of the GME. An important rise in prices reduces the competitiveness of the affected firm within the set of firms that focus on the same

[60] In theory, the upstream effect on prices is indeterminate. When a firm experiences a negative productivity shock, its selling price rises and output decreases. This fact reduces the demand for inputs and therefore, the price at which suppliers are willing to sell their products. Nevertheless, a productivity shock also raises firms' costs due to the reduction in the factors of production efficiency. This aspect, in turn, increases the demand for inputs and raises the price set by suppliers. Consequently, the variation in the prices set by suppliers are ambiguous.

market segment, thereby decreasing its chances of selling its product and earning a profit. This effect may diffuse in the production network through the buying-selling transactions and endanger the profitability of other firms. To examine this phenomenon, we calculate the percentage variation in the frequency with which firms earn non-negative profits in the initial period. This frequency gives an idea of a firm's probability of earning a positive profit and engaging in trade relations with its buyers.

Variations in this indicator are shown in Figure 2.14.a, where the red bars correspond to negative values and the blue bars, positive ones. The same color coding is used for nodes in the network represented in Figure 2.14.b, where the delineated path corresponds to the firms that exhibit the largest negative variations. In inspecting both figures, it is observed that the shock on firm j_9 generates an upstream cascade effect that affects firms' profits, and which is propagated only towards the regular suppliers in the network.

Figure 2.14. The GME in the alternative scenario. Prod. Shock of 20% on firm 9. Variations in the Frequency of non-negative profits. Initial period.



Note. Figure a) is a bar diagram that shows the percentage variation in the frequency with which firms earn non-negative profits. This frequency is computed based on the empirical distribution function of the GME before and after the productivity shock. The red-blue color gradation indicates the sign of the variation. The color red indicates a reduction in the frequency, while a blue coloration denotes an increase in the frequency. Figure b) replicates this color grading on the nodes in the production network and outlines a path with the nodes that exhibit the largest negative variation.

As can be observed, the shock that impacts firm j_9 reduces its chances of obtaining positive profits by 8.0%. On top of that, it also reduces this possibility for its regular supplier j_5 by 6.2%, and for the regular supplier of the latter j_1 by 5.2%. These effects are accompanied by positive effects on the firms that tend to replace the affected supplier in the same linkage level. For example, after the shock that affects firm j_9, competing firms j_7 and j_8 increase their chances of earning positive profits by 1.3% and 4.9%, respectively.

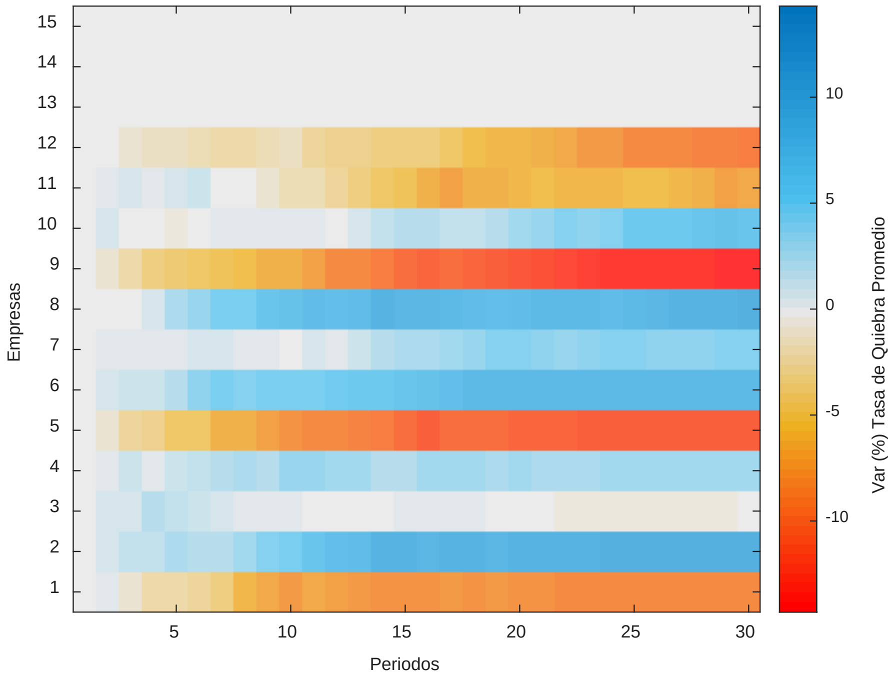
The reason why this upstream cascade effect propagates exclusively to the regular suppliers can be explained by the high frequency with which these suppliers engage in transactions in the network. Regular suppliers earn a profit on most occasions in which buyers choose to transact with them, which is why if the buyers make no profits (because their selling prices are less competitive), they don't either. In other words, if a supplier usually sells its product to some buyer in the network, all that happens to this buyer has an impact on the supplier's profits (and consequently on the profits of those suppliers who regularly supply the latter). It must be emphasized that the shock's diffusion favors the rival suppliers' competitiveness, who, on the other hand, increase their chances of trading in the network and therefore, of obtaining a profit.

Appendix G.2. shows that the upstream cascade effect on the regular suppliers' profits is present in all alternative scenarios, except for those scenarios where the productivity shock affects firms that produce final consumption goods. In these scenarios, the cascade effect is sometimes diffused through the rival suppliers, producing variations which are milder than those observed in the first levels of linkages.

The upstream diffusion of the productivity shock over firms' profits during the initial period is critical for understanding how firms go bankrupt over the course of time. As may be recalled from the model's design, a firm has a higher probability of going bankrupt as the number of periods where it registered zero profits increases. Along these lines, if a shock's influence on the mechanism for earning profits is known, then its effect on firm bankruptcy can be explained. To analyze this effect, we turn to Figure 2.15. This figure shows the variation in the average bankruptcy rate of firms in the production network through a heat map that uses an inverted red-blue color gradation. The color red signals a positive variation (i.e. the chances of a firm going bankrupt increase) while the color blue indicates that this variation is negative (i.e. the odds of a firm remaining in the network increase). The higher the intensity of each color, the greater the variation in the firm's bankruptcy rate (its absolute value) over a given period.

Broadly speaking, we observe that the productivity shock on firm j_9 exacerbates the bankruptcy of the regular supplier found upstream. First, this shock causes an increase in firm's j_9 chances of going bankrupt, starting at period 3, until reaching the increase in its rate of bankruptcy of 14% by the end of the 30 time periods. This effect is propagated from supplier to supplier, with lessened intensity. For example, it may be observed that firms j_5 and j_1 augment their odds of going bankrupt in periods 3 and 5, but with an increase in their bankruptcy rates of 10% and 8% by the end of the 30 periods, respectively. A negative effect on firms j_{11} and j_{12}, firm j_9's buyers, is also observed, albeit milder.

Figure 2.15. Firm bankruptcy in the alternative scenario. Prod. Shock of 20% on firm 9. Variation on the Average Bankruptcy Rate by firm. 30 time periods.



Note. This figure shows the percentage variation in the average bankruptcy rate of firms over a set of 1,000 random trials in a time horizon of 30 periods. Each row represents a firm and each column, a time period. The red-blue color gradation specifies the sign and magnitude of the variation. If colored red, then there is an increase in the rate of bankruptcy. The higher the intensity of the color red, the larger the rise in the bankruptcy rate. On the contrary, if colored blue, then there is a decrease in the rate of bankruptcy. The more intense the blue color is, the larger the decrease in the rate of bankruptcy.

In this figure it is worth noting that there are firms which experience an increase in their odds of remaining in the industry due to their rival supplier status. This is the case of firms j_2, j_4, j_6, j_7, j_8, which augment their permanence in the network by replacing regular suppliers that were displaced by the productivity shock.

As was mentioned before, the upstream cascade effect on profits in the initial period helps to understand the origin of the upstream cascade effect on the bankruptcy of regular suppliers. A productivity shock affects a firm by increasing its selling prices, which reduces its chances of trading in the network and gaining positive profits in the initial period. This effect spreads from supplier to supplier in a way that regular suppliers also experience a reduction in their chances of earning a profit. In this way, the entire chain of firms, starting at the firm that was hit by the shock until the last of the affected suppliers, will experience a higher probability of going bankrupt in the next period.

If in the next period one of these firms goes bankrupt, the economic activity of this firm's regular suppliers will contract considerably (since, losing their buyer, a large part of their sales is lost as well), further restricting their chances of earning a profit and thus, remaining in the network. The same will be experienced by the rest of regular suppliers located upstream, due to the cascade effect on firms' profits which happens in that point in time. This process occurs successively from period to period, as firms exit the industry and further condition the profits of those firms that remain in the production network.

Appendix G.3. shows the upstream cascade effect on firm bankruptcy in all alternative scenarios. As was the case in the productivity shock on firm j_9, in some scenarios, the shock also produces effects on some buyers; however, these effects have lower magnitude and are unstable over time. It must be underlined that in these results there exists some sort of intertemporal substitution between regular suppliers and rival suppliers in the network, since the latter become less competitive as a result of the shock.[61]

---

[61] The intertemporal substitution between regular and rival suppliers during the process of firm bankruptcy stabilizes the number of firms that remain in the network in relation to the base scenario, consequently, no

2.5 Conclusions

The diffusion of microeconomic shocks in production networks stands as one of the most developed research fields in the analysis of economic networks, in the last decade. Its objective is delimited by the study of cascade effects, in other words, the propagation of productivity shocks towards the chain of buyers or suppliers in the network, through intersectoral input-output relations.

In the same line, diverse theoretical work focused on the theory of general equilibrium has been developed, with the widespread use of assumptions such as: markets with perfect competition, representative firms, exogeneous networks, permanent firms (with no possibility of bankruptcy), and Cobb-Douglas technologies and preferences. These assumptions incorporate strong restrictions regarding the production network's behavior and reduce the complexity of the economic system under study. As such, they may lessen the propagation of microeconomic shocks or restrict its effects to certain firms in the network.

The present document makes a special contribution to this theoretical framework. It analyzes the propagation of microeconomic shocks in production networks with imperfect competition and the possibility of firm bankruptcy. Here, firms that produce intermediate goods have ability to set the price of their output, considering that their buyers exhibit a probabilistic choice behavior. This behavior assumes that buyers assign a probability to each of the transactions with their potential suppliers, a probability that emerges from the trade-off they face between profit and risk, due to the lack of information regarding the transactions' terms. Determined this way, the intermediate goods prices, together with other economic variables, help us define what we call the General Market Equilibrium (GME). On the other hand, firm bankruptcy is modelled using a stochastic process in which each time period is determined by the GME of the firms that survived up to that period. Here, a firm's probability of remaining in the industry is determined by the number of periods where the firm obtained a positive profit; the larger this number, the higher this probability.

---

significant variations in the macroeconomic aggregates are to be expected. For further detail see Appendix G.4.

Under these circumstances, a productivity shock on a firm produces several effects. First, there are two effects on the selling prices in the initial period: a horizontal effect and a downstream cascade effect. The former effect increases the selling prices of firms that compete with the impacted firm, while the latter effect raises the selling prices from buyer to buyer in the network. These effects stem from the price setting behavior and the rivalry between firms. Second, there is an upstream cascade effect on firms' profits in the initial period. This effect is produced by the loss of competitiveness in the affected firm and is propagated from regular supplier to regular supplier in the network, due to the regularity with which these firms engage in buying-selling transactions. Third, there is an upstream cascade effect on firm bankruptcy over the course of time. Here, regular suppliers go bankrupt period after period as a consequence of the upstream cascade effect on profits, which is replicated successively throughout the time horizon and more intensely when some firm exits permanently the network.

Although these results are supported by numerical simulations performed on a small non-complex productive network (a network structured uniformly by levels of linkages without cycles), they constitute an important display of the various indirect effects generated by a microeconomic shock when certain fundamental assumptions from the competitive equilibrium and the analysis of networks are relaxed. In this sense, it may be relevant to explore other problems which go hand in hand with price setting and firm bankruptcy, such as regional market segmentation, firm centrality and capital investment.

Conclusions

The chapters in this thesis investigate two big topics about networks in economy: the interaction between accountants and taxpayers in firm's tax reporting when any of them (or both) are notified by tax administration, and the diffusion of microeconomic shocks through input-output interrelations in economies with imperfect competition and firms' bankruptcy. Even though these chapters have a common background (i.e. networks), they have a different scope and methodology. The first topic about the participation of accountants and taxpayers in firm's tax reporting is fully empirical and focus on impact evaluation of deterrent notifications on income tax. The second topic about shock diffusion is fully theoretical. It develops a microeconomic model of productive networks and supports its findings with numeric simulations.

The first chapter analyzes the effect of electronic deterrent notifications on accountants, through an experiment in Ecuadorian tax system with the cooperation of tax administration. This experiment sent five different kinds of deterrent notifications to accountants and taxpayers prior to the reporting deadline of fiscal year 2015 via the tax box system. Using a simple regression model and information of corporate income tax for microenterprises, it was shown that simultaneous notifications on both accountants and taxpayers were the only treatmeant that increased significantly firms' declared income tax. They were even more effective at improving firms' declared tax than notifications on accountants only. Futhermore, it was shown that penalty notifications on accountants, rather than taxpayers only, were the most significant treatment at increasing firms' declared revenue, however they did not generate a significant impact on declared tax due to a cost overreporting mechanism.

All these results demostrate that there is a systemic relationship between accountants and taxpayers that could explain how firms evade taxes and how they react when any of the parties is notified by the tax administration. In fact, these findings provide initial insight to microeconomic theories about tax evasion through agency problems, such as the Crocker and Slemrod's model.

The second chapter examines how a productive shock could be propagated in productive networks with imperfect markets and bankruptcy. To do so, it was built a microeconomic network model with three kind of agents: intermediate good firms, final-good firms and a representative household. Intermediate good firms can set their selling prices, taking into account that buyers have a probabilistic choice behavior. This behavior supposes the buyers assign a probability to each transaction they make with their suppliers, as a result of the trade-off between expect profits and risk of loss they face due to the lack of information about transactions (e.g. quality product). Final-good firms and the representative household are price-takers. This economic system can be used to define what is called in this research the General Market Equilibrium (GME), which represents a kind of Nash equilibrium within a structure of ex-post supply-demand equilibrium. Furthermore, the firm bankruptcy is represented by a stochastic process in which each period is determined by the GME of the firms that survived until that moment.

Based on this model, the simulation of productive shocks shows different cascade effects on prices and economic flows of the productive network. First, there are two types of price effects in the initial period: a horizontal effect and a downstream cascade effect. The former increases selling prices for the firms that belongs to the competition of the impacted firm, and it is produced by the rivalry between firms. The latter increases the selling prices from buyer to buyer and it is caused by the increase in marginal costs after the shock. Second, there is an upstream effect on firms' profit in the initial period, due to the loss of competitiveness in the impacted firm and the contraction in the production of regular suppliers (i.e. those suppliers that commonly engage in buying-selling transations and have the lowest risk). Finally, it is found an upstream cascade effect on firm bankruptcy that is propagated exclusively to regular suppliers in the network over time. In other words, if the firm that received the shock goes bankrupt, their regular suppliers, the regular suppliers of these suppliers, and so on, will most likely go bankrupt in the next periods. This effect is generated as a result of the effects described above, and is replicated each period more intensely as firms exits the network.

Although the simulations of the model are limited to a small non-complex productive network, they provide a preliminary view about shocks diffusion when there are market failures and firm bankruptcy. As such, these results complement Acemolgu's findings, which instead show that productivity shocks in productive networks with perfect competition could generate downstream cascade effects.

Appendix

A. Notifications

A.1. Treatment 1. Accountant placebo notification

INTERNAL RENTS SERVICE

Dear

<Name>

Accountant

Declare income tax on time:

The Internal Rents Service is pleased to inform that the deadline for the income tax declaration for the fiscal year 2015 began on February 1, 2016, and will end in the case of natural persons on 28 March 2016, and in the case of companies on April 28, 2016; according to the ninth digit of the RUC.

Therefore, you are asked to review timely the values that will be reported and to prepare the accounting and tax documentation of taxpayers for whom you work as an accountant.

Sincerely,

INTERNAL RENTS SERVICE

A.2. Treatment 2. Accountant penalty notification

INTERNAL RENTS SERVICE

Dear

<Name>

Accountant

Declare income tax on time:

The Internal Rents Service is pleased to inform that the deadline for the income tax declaration for the fiscal year 2015 began on February 1, 2016, and will end in the case of natural persons on 28 March 2016, and in the case of companies on April 28, 2016; according to the ninth digit of the RUC.

Therefore, you are asked to review timely the values that will be reported and to prepare the accounting and tax documentation of taxpayers for whom you work as an accountant.

Avoid penalties:

In your status as accountant, according to article 101 of Internal Tax Regime Law, you are liable for the accuracy and reliability of the data reported in the firm's declaration.

The Internal Revenue Service has the legal authority, as well as the technical and computer tools, to verify the fulfilment of tax obligations. In the case to check any infringement, we will initiate the appropriate legal actions.

Tax fraud, for your own or third parties benefit, is punished with imprisonment of three to seven years according Article 298 of the Criminal Integral Organic Code.

Sincerely,

INTERNAL RENTS SERVICE

A.3. Treatment 3. Accountant risk notification

INTERNAL RENTS SERVICE

Dear

<Name>

Accountant

Declare income tax on time:

The Internal Rents Service is pleased to inform that the deadline for the income tax declaration for the fiscal year 2015 began on February 1, 2016, and will end in the case of natural persons on 28 March 2016, and in the case of companies on April 28, 2016; according to the ninth digit of the RUC.

Therefore, you are asked to review timely the values that will be reported and to prepare the accounting and tax documentation of taxpayers for whom you work as an accountant.

Avoid tax controls:

The Internal Revenue Service, according to information from the second half of 2015, has detected that you carry out the accounting of <###> taxpayers.

The calculation of income tax is based on the data recorded in the accountancy, and to the extent that the accounting technique is applied correctly, the tax reporting will be appropriate. Therefore, as an accountant, you must ensure that financial statements fairly present the economic and financial situation of each of these taxpayers, and also support documents meet all legal requirements for issuing them.

If there are irregularities, we will start the appropriate legal action.

Sincerely,

INTERNAL RENTS SERVICE

A.4. Treatment 4. Accountant-Taxpayer risk notification (notification for accountant)

INTERNAL RENTS SERVICE

Dear

<Name>

Accountant

Declare income tax on time:

The Internal Rents Service is pleased to inform that the deadline for the income tax declaration for the fiscal year 2015 began on February 1, 2016, and will end in the case of natural persons on 28 March 2016, and in the case of companies on April 28, 2016; according to the ninth digit of the RUC.

Therefore, you are asked to review timely the values that will be reported and to prepare the accounting and tax documentation of taxpayers for whom you work as an accountant.

Avoid tax controls:

The Internal Revenue Service, according to information from the second half of 2015, has detected that you carry out the accounting of <###> taxpayers.

The calculation of income tax is based on the data recorded in the accountancy, and to the extent that the accounting technique is applied correctly, the tax reporting will be appropriate. Therefore, as an accountant, you must ensure that financial statements fairly present the economic and financial situation of each of these taxpayers, and also supporting documents meet all legal requirements for issuing them.

If there are irregularities, we will start appropriate legal actions.

For your knowledge, taxpayers for whom you work as an accountant will be notified of all information shown here.

Sincerely,

INTERNAL RENTS SERVICE

A.5. Treatment 4. Accountant-Taxpayer risk notification (notification for taxpayer)

INTERNAL RENTS SERVICE

Dear

<Name>

Taxpayer

Declare income tax on time:

The Internal Rents Service is pleased to inform that the deadline for the income tax declaration for the fiscal year 2015 began on February 1, 2016, and will end in the case of natural persons on 28 March 2016, and in the case of companies on April 28, 2016; according to the ninth digit of the RUC.

Therefore, you are asked to review timely the values that will be reported and to prepare the accounting and tax documentation used for the declaration.

Avoid tax controls:

The Internal Revenue Service, according to information from the second half of 2015, has detected that your accountant carries out the accounting of <###> taxpayers.

The calculation of income tax is based on the data recorded in the accountancy, and to the extent that the accounting technique is applied correctly, the tax reporting will be appropriate. Therefore, your accountant must ensure that financial statements fairly present the economic and financial situation of each of these taxpayers, and also supporting documents meet all legal requirements for issuing them.

If there are irregularities, we will start appropriate legal actions.

For your knowledge, your accountant will be notified of all information shown here.

Sincerely,

INTERNAL RENTS SERVICE

A.6. Treatment 5. Taxpayer penalty notification

INTERNAL RENTS SERVICE

Dear

<Name>

Taxpayer

Declare income tax on time:

The Internal Rents Service is pleased to inform that the deadline for the income tax declaration for the fiscal year 2015 began on February 1, 2016, and will end in the case of natural persons on 28 March 2016, and in the case of companies on April 28, 2016; according to the ninth digit of the RUC.

Therefore, you are asked to review timely the values that will be reported and to prepare the accounting and tax documentation used for the declaration.

Avoid penalties:

In your status as legal representative, according to article 101 of Internal Tax Regime Law, you are liable for the accuracy and reliability of the data reported in the firm's declaration.

The Internal Revenue Service has the legal authority, as well as the technical and computer tools, to verify the fulfilment of tax obligations. In the case to check any infringement, we will initiate the appropriate legal actions.

Tax fraud, for your own or third parties benefit, is punished with imprisonment of three to seven years according Article 298 of the Criminal Integral Organic Code.

Sincerely,

INTERNAL RENTS SERVICE

B. Impact differences between treatments

B.1. Impact Differences on declared total revenue. Fiscal year 2015

	T1. Acc. Placebo notification	T2. Acc. Penalty notification	T3. Acc. Risk notification	T4. Acc-Txp. Risk notification	T5. Txp. Penalty notification
T1. Acc. Placebo notification					
T2. Acc. Penalty notification	0,059**				
	(0,036)				
T3. Acc. Risk notification	0,025	-0,034			
	(0,405)	(0,242)			
T4. Acc-Txp. Risk notification	0,077**	0,018	0,052		
	(0,048)	(0,643)	(0,190)		
T5. Txp. Penalty notification	0,007	-0,052	-0,018	-0,070	
	(0,844)	(0,146)	(0,629)	(0,112)	

Note: Each cell of this matrix shows the difference between one impact treatment (row) to other (column) on total revenue relative change in the post-treatment period (fiscal year 2015). All covariable's sets were included for the estimation (firm's variables, accountant's variables and time variables). Pvalors are in parenthesis. Robust standard errors were corrected by accountant cluster. *, **, *** denote significance at 10%, 5%, and 1% levels, respectively.

B.2. Impact Differences on declared total cost. Fiscal year 2015

	T1. Acc. Placebo notification	T2. Acc. Penalty notification	T3. Acc. Risk notification	T4. Acc-Txp. Risk notification	T5. Txp. Penalty notification
T1. Acc. Placebo notification					
T2. Acc. Penalty notification	0,052*				
	(0,074)				
T3. Acc. Risk notification	0,025	-0,027			
	(0,426)	(0,388)			
T4. Acc-Txp. Risk notification	0,024	-0,028	-0,001		
	(0,540)	(0,475)	(0,978)		
T5. Txp. Penalty notification	0,005	-0,047	-0,020	-0,019	
	(0,896)	(0,203)	(0,601)	(0,668)	

Note: Each cell of this matrix shows the difference between one impact treatment (row) to other (column) on total cost relative change in the post-treatment period (fiscal year 2015). All covariable's sets were included for the estimation (firm's variables, accountant's variables and time variables). Pvalors are in parenthesis. Robust standard errors were corrected by accountant cluster. *, **, *** denote significance at 10%, 5%, and 1% levels, respectively.

B.3. Impact Differences on declared income tax. Fiscal year 2015

	T1. Acc. Placebo notification	T2. Acc. Penalty notification	T3. Acc. Risk notification	T4. Acc-Txp. Risk notification	T5. Txp. Penalty notification
T1. Acc. Placebo notification					
T2. Acc. Penalty notification	0,046				
	(0,161)				
T3. Acc. Risk notification	0,034	-0,013			
	(0,320)	(0,648)			
T4. Acc-Txp. Risk notification	0,110***	0,063*	0,076**		
	(0,004)	(0,054)	(0,028)		
T5. Txp. Penalty notification	0,073*	0,026	0,039	-0,037	
	(0,063)	(0,434)	(0,264)	(0,339)	

Note: Each cell of this matrix shows the difference between one impact treatment (row) to other (column) on income tax relative change in the post-treatment period (fiscal year 2015). All covariable's sets were included for the estimation (firm's variables, accountant's variables and time variables). Pvalors are in parenthesis. Robust standard errors were corrected by accountant cluster. *, **, *** denote significance at 10%, 5%, and 1% levels, respectively.

C. Preference approach to probabilistic choice problem

Assume that buyer k has the preferences $\succcurlyeq^{(k)}$ defined on the vector $V = (\Pi, \mathcal{E})$, where Π is the expected profit and $\mathcal{E}$ is the expected security obtained from inputs. Here, security represents the expected value that a catastrophe loss does not occur, that is, it is a variable contrary to risk. According to Figure 2.4, the values of $\Pi, \mathcal{E}$ for the regular supplier j and the rival supplier $\bar{j}$ can be calculated as:

$$\Pi_j = (1 - \phi_j)\pi_j + \phi_j \varepsilon_j, \qquad \Pi_{\bar{j}} = (1 - \phi_{\bar{j}})\pi_{\bar{j}} + \phi_{\bar{j}}\varepsilon_{\bar{j}}$$
$$\mathcal{E}_{\mathrm{j}} = (1 - \phi_j)\varepsilon_j, \qquad \mathcal{E}_{\bar{\mathrm{j}}} = (1 - \phi_{\bar{j}})\varepsilon_{\bar{j}}$$

As it will be recalled, π_j, ε_j y ϕ_j are the positive profits, negative profits and risk of loss for the regular supplier, respectively; while $\pi_{\bar{j}}, \varepsilon_{\bar{j}}$ are the level of profits and losses obtained with this supplier, respectively, and $\phi_{\bar{j}}$ is the risk of incurring losses.

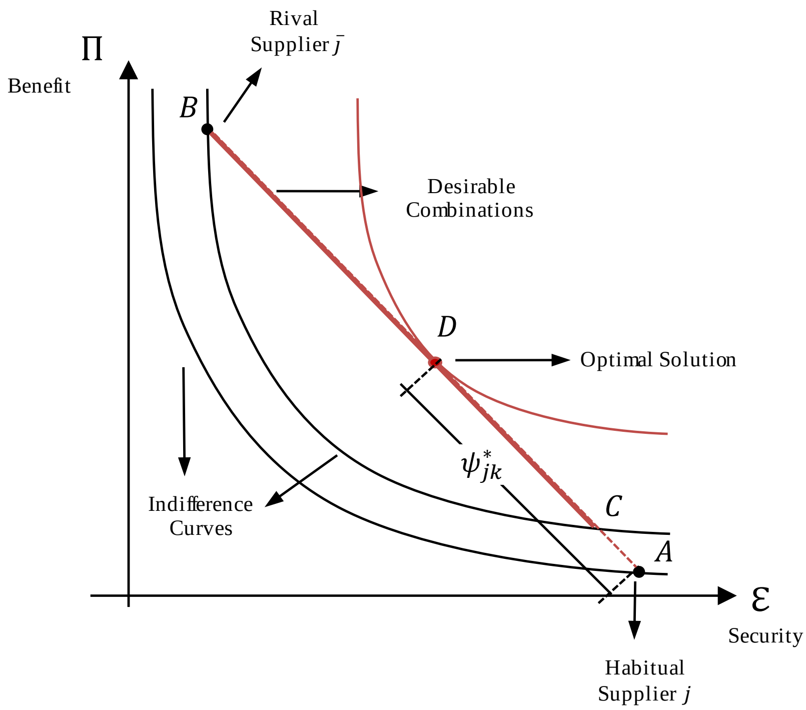
Assume that the preferences $\succcurlyeq^{(k)}$ are rational, continuous and convex, so there is a utility function $U^{(k)}$ quasiconcave that represents them. This function measures the buyer's satisfaction, based on the expected profit and the security offered by his suppliers.

Because the preferences are convex, the buyer will always prefer to make a transaction that has the best terms (benefit and security) of the proposals from both suppliers, instead of making a transaction with a single supplier, since this fact will allow him to increase his utility. In this sense, the probabilistic choice of the buyer k represents the proportion at which the proposals of the regular supplier j and the rival $\bar{j}$ are combined, in such a way that his utility is maximized. Formally,

$$\begin{aligned}
&\max_{\psi_{jk}} U^{(k)}(\widehat{\Pi}, \hat{\mathcal{E}}) \\
&s.r \\
&\widehat{\Pi} = \psi_{jk}\Pi_j + (1 - \psi_{jk})\Pi_{\bar{j}} \\
&\hat{\mathcal{E}} = \psi_{jk}\mathcal{E}_{\mathrm{j}} + (1 - \psi_{jk})\mathcal{E}_{\bar{\mathrm{j}}} \\
&0 \le \psi_{jk} \le 1
\end{aligned}$$

The solution to this problem is illustrated in the following figure, when the rival supplier offers the greatest utility to the buyer, that is, $U^{(k)}(\Pi_{\bar{j}}, \mathcal{E}_{\bar{j}}) > U^{(k)}(\Pi_j, \mathcal{E}_j)$. Point A shows the proposal of the regular supplier (lower profit, greater security), while point B shows the proposal of the rival supplier (greater profit, lower security). The indifference curves that pass over these points delimit the segment BC, which contains all combinations that gives the buyer a greater preference. The most preferred combination is reached at point D where a new indifference curve is tangent to the segment BC. In this way, the probability ψ_{jk}^{*} that maximizes the buyer's utility is determined by the proportion that segment DC represents within segment BA.

Figure. Solution to probabilistic choice problem under a preference approach



Note. This figure shows the solution to probabilistic choice problem under a preference approach, when the rival supplier offers greater utility to the buyer, that is, $U^{(k)}(\Pi_{\bar{j}}, \mathcal{E}_{\bar{j}}) > U^{(k)}(\Pi_j, \mathcal{E}_j)$. The abscissa axis represents the expected security Ɛ and the ordinate axis represents the expected benefit Π. Point A represents the proposal of the regular supplier (lower profit, greater security), and point B represents the proposal of the rival supplier (greater profit, lower security). Segment BC represents the set of combinations that are preferred to points A and B, reaching the most preferred combination at point D. The probability ψ_{jk} is determined by the proportion that segment DC represents within segment BA.

D. Demand functions with CES technology/preferences

D.1 Conditional demand functions and minimum cost functions

All firms of the production network are price takers when they demand inputs. Their objective is to minimize the total cost, subject to a technological restriction CES with constant returns of scale and fixed capital.

Formally, each $j \in J$ firm faces the following decision problem:

$$\min_{z_{ji}, l_j} c_j = \sum_{i \in I^{\text{m}}} p_i \, z_{ji} + w l_j + r \bar{k}_j$$

$$y_j = A_j \left(\sum_{I^{\text{m}}} \alpha_{ji} z_{ji}^{\rho_j} + \alpha_{jl} l_j^{\rho_j} + \alpha_{jk} \bar{k}_j^{\rho_j} \right)^{\frac{1}{\rho_j}}$$

where $y_j \in \mathbb{R}_+$ is the production of the firm j, $z_j = \left(z_{j1}, \dots, z_{jM^{\text{m}}} \right) \in \mathbb{R}_+^{M^{\text{m}}}$ is the vector of input demand, $l_j \in \mathbb{R}_+$ is labor demand, $\bar{k}_j$ is the demand for fixed capital, $p = (p_1, \dots, p_{M^{\text{m}}}) \in \mathbb{R}_+^{M^{\text{m}}}$ is the vector of input prices, w is the wage and r is the capital return[62]. $A_j > 0$ is a productivity constant; $\alpha_{ji}, \alpha_{jl}, \alpha_{jk} > 0$ are participation coefficients of input demand, and $\rho \leq 1$ is a parameter related to the elasticity of substitution between factors.

The solution to this problem using the method of Lagrange multipliers produces the following conditional demand functions:

$$z_{ji}\left(y_j, p, w \right) = \left(\frac{y_j^{\rho_j}}{A_j} - \alpha_{jk} \bar{k}_j^{\rho_j} \right)^{\frac{1}{\rho_j}} \frac{\left(\frac{\alpha_{ji}}{p_i} \right)^{\frac{1}{1-\rho_j}}}{\left(\sum_{i' \epsilon I^{\text{m}}} p_{i'} \left(\frac{\alpha_{ji'}}{p_{i'}} \right)^{\frac{1}{1-\rho_j}} + w \left(\frac{\alpha_{jl}}{w} \right)^{\frac{1}{1-\rho_j}} \right)^{1/\rho_j}}, \qquad \forall i \epsilon I^{\text{m}}$$

$$l_j\left(y_j, p, w \right) = \left(\frac{y_j^{\rho_j}}{A_j} - \alpha_{jk} \bar{k}_j^{\rho_j} \right)^{\frac{1}{\rho_j}} \frac{\left(\frac{\alpha_{jl}}{w} \right)^{\frac{1}{1-\rho_j}}}{\left(\sum_{i' \epsilon I^{\text{m}}} p_{i'} \left(\frac{\alpha_{ji'}}{p_{i'}} \right)^{\frac{1}{1-\rho_j}} + w \left(\frac{\alpha_{jl}}{w} \right)^{\frac{1}{1-\rho_j}} \right)^{\frac{1}{\rho_j}}}$$

On the other hand, the minimum cost function is:

---

[62] It should be noted that the vector p shows the price of the inputs after the firm selected their suppliers in the realization of $\omega \in \Omega$, so it is related to the set of vectors p_{-j}^{m} in the problem (4).

.

$$c_j(y_j, p, w) = \left(\frac{y_j^{\rho_j}}{A_j} - \alpha_{jk}\bar{k}_j^{\rho_j}\right)^{\frac{1}{\rho_j}} \left(\sum_{i' \epsilon I^{\mathrm{m}}} p_{i'} \left(\frac{\alpha_{ji'}}{p_{i'}}\right)^{\frac{1}{1-\rho_j}} + w\left(\frac{\alpha_{jl}}{w}\right)^{\frac{1}{1-\rho_j}}\right)^{(\rho_j-1)/\rho_j} + r\bar{k}_j$$

D.2. Marshallian demand and labor supply functions of the representative household

Suppose the following utility CES with preference to leisure:

$$u(x(\omega), l_h(\omega)) = \left(\sum_{i\epsilon I^{\mathrm{g}}} \gamma_i x_i^{\eta}(\omega) + \gamma_l\left(\bar{L} - l_h(\omega)\right)^{\eta}\right)^{\frac{1}{\eta}}$$

where $x(\omega)$ is the vector of input demand, $l_h(\omega)$ is the labor supply for each realization of the productive network $\omega \in \Omega$. $\gamma_i, \gamma_l > 0$ are consumption and leisure coefficients, respectively; and $\eta \leq 1$ is a parameter related to the elasticity of substitution between final goods and labor.

Therefore, the solution of problem (7) by the method of Lagrange multipliers produces the following functions of Marshallian demand:

$$x_i(\omega, p^{\mathrm{g}}(\omega), w(\omega), \pi(\omega)) =$$

$$\left(\frac{\gamma_i}{p_i^{\mathrm{g}}(\omega)}\right)^{\frac{1}{1-\eta}} \frac{\pi(\omega) + w(\omega)\bar{L}}{\sum_{i'\epsilon I^{\mathrm{g}}} p_{i'}^{\mathrm{g}}(\omega)\left(\frac{\gamma_{i'}}{p_{i'}^{\mathrm{g}}(\omega)}\right)^{\frac{1}{1-\eta}} + w(\omega)\left(\frac{\gamma_l}{w(\omega)}\right)^{\frac{1}{1-\eta}}}, \qquad \forall i\epsilon I^{\mathrm{g}} \ \forall \omega \in \Omega$$

On the other hand, the optimal labor supply is defined by:

$$l_h(\omega, p^{\mathrm{g}}(\omega), w(\omega), \pi(\omega)) =$$

$$\bar{L} - \left(\frac{\gamma_l}{w(\omega)}\right)^{\frac{1}{1-\eta}} \frac{\pi(\omega) + w(\omega)\bar{L}}{\sum_{i'\epsilon I^{\mathrm{g}}} p_{i'}^{\mathrm{g}}(\omega)\left(\frac{\gamma_{i'}}{p_{i'}^{\mathrm{g}}(\omega)}\right)^{\frac{1}{1-\eta}} + w(\omega)\left(\frac{\gamma_l}{w(\omega)}\right)^{\frac{1}{1-\eta}}}, \qquad \forall \omega \in \Omega$$

E. Proof of the existence for the General Market Equilibrium.

The existence of the General Market Equilibrium is demonstrated in 5 steps. First, it is proved that the solution to the system of equations $H(x(\omega), y(\omega), p^{\mathfrak{g}}(\omega), w(\omega), p^{\mathfrak{m}})$ for each scenario $\omega \in \Omega$ given the prices $p^{\mathfrak{m}}$ exists. Second, it is proved that prices $p^{\mathfrak{m}}$ belong to a compact and convex set. Third, it is proved that the expected profit for firms that produce intermediate goods $E\left[\pi_j / p_j^{\mathfrak{m}}, p_{-j}^{\mathfrak{m}}\right]$ is a continuous function. Fourth, it is proved that this function is concave. Finally, based on the result of the previous steps, it is proved that there is a price matrix $p^{\mathfrak{m}*}$ that solves the reaction functions $p_j^{\mathfrak{m}*} = F_j\left(p_{-j}^{\mathfrak{m}*}, W, Y_{-j}\right), \forall j \in J^{\mathfrak{m}}$.

To simplify the demonstration, it is assumed that the utility of representative household is Cobb-Douglas without preference on leisure (i.e. there is a fixed total labor). In addition, firms are supposed to have Marx-Leontief technology for input demand. For firms $j \in J^{\mathfrak{m}}$, labor is requiered in fixed proportions, while for firms $j \in J^g$ labor is required with a decreasing marginal productivity. It is also assumed that each final good is produced by a representative firm. Finally, it is assumed that the function that measures the trade-off between profit and risk in the probabilistic choice problem is linear.

E.1. The solution of the system $\boldsymbol{H(x(\omega), y(\omega), p^{\mathfrak{g}}(\omega), w(\omega), p^{\mathfrak{m}})}$ exists

Let $\omega \in \Omega$ be a realization of the productive network. Let $p^{\mathfrak{m}} \in \mathbb{R}_+^N \times \mathbb{R}_+^{N^{\mathfrak{m}}}$ be the prices of intermediate goods in the production network. These variables will be considered fixed in this first part of the demonstration. Because firms have Marx-Leontief technology, the demand for each input is a fixed proportion of firms' production. In formal terms:

$$\forall j \in J, \forall i \in b(j), \;\; z_{ji}(\omega) = \alpha_{ji} y_j(\omega)$$

where z_{ji} is the demand of input i for firm j, α_{ji} is the technical coefficient of the input i for firm j, and y_j is the total production of firm j in the realization ω. On the other side, by the ex-post local equilibrium in the intermediate good market, we have:

$$\forall j \in J^{\mathfrak{m}}, \qquad y_j(\omega) = \sum_{k \in \Delta_j^+(\omega)} z_{k,a(j)}(\omega)$$

Consequently, the production of an intermediate good firm can be represented as a linear combination of the production of its buyers.

$$\forall j \in J^{\mathfrak{m}}, \qquad y_j(\omega) = \sum_{k \in \Delta_j^{+}(\omega)} \alpha_{k,a(j)} y_k(\omega)$$

If this property is applied from buyer to buyer in the productive network, then it can be concluded that the production of any firm can be represented as a linear combination of the supply of final consumption goods. In other words:

$$\forall j \in J^{\mathfrak{m}}, \qquad y_j(\omega) = \sum_{i \in I^{\mathfrak{g}}} \theta_i(\omega) \hat{y}_i(\omega) \tag{1}$$

where $\theta_i(\omega)$ is a constant that depends on the realization ω, and $\hat{y}_i(\omega)$ is the production of the representative firm i. This equation states an important characteristic in the solution of the system $H(x(\omega), y(\omega), p^{\mathfrak{g}}(\omega), w(\omega), p^{\mathfrak{m}})$. If the production of the final-good firms is determined, then the production of any intermediate good firm in the network will be too. In this sense, it is enough to demonstrate the equilibrium in the final-good market and the labor market to solve the system.

First, we will study the equilibrium in final-good market. On the supply side, the representative firm $i \in I^{\mathfrak{g}}$ carries out its activity in competitive markets. That is, it decides how much to produce in order to maximize its profits given market prices. This problem is shown below:

$$\max_{y_i^{\mathfrak{g}}(\omega)} \; p_i^{\mathfrak{g}}(\omega)\hat{y}_i(\omega) - \sum_{k \in b(i)} \alpha_{ik} \hat{y}_i(\omega) p_{ki}^{\mathfrak{m}} - w l_i\big(\hat{y}_i(\omega)\big)$$

where $p_i^{\mathfrak{g}}(\omega)$ is the price of final good i, α_{ik} is the technical coefficient of the input k for firm i, $p_{ki}^{\mathfrak{m}}$ is the price of input k requiered by the firm i, w is the wage and $l_i\big(\hat{y}_i(\omega)\big)$ is the labor demand function such that:

$$\forall i \in I^{\mathfrak{g}}, \qquad l_i\big(\hat{y}_i(\omega)\big) = \hat{y}_i(\omega)^{\vartheta_i}$$

where ϑ_i is a coefficient related to labor marginal productivity. It is assumed that $\vartheta_i > 1$ in order to assume a diminishing marginal productivity. The solution to this problem is:

$$\hat{y}_i(\omega) = \left(\frac{p_i^{\mathfrak{g}}(\omega) - \sum_k \alpha_{ik} p_{ki}^{\mathfrak{m}}}{w\vartheta_i}\right)^{\frac{1}{\vartheta_i - 1}} \tag{2}$$

It should be noted that this function exists whenever $p_i^{\mathfrak{g}}(\omega) > \sum_k \alpha_{ik} p_{ki}^{\mathfrak{m}}$. This restriction could be called as activity condition and implies that the representative firm $i \in I^{\mathfrak{g}}$ produces if the price is higher than the input costs per unit.

On the demand side, the representative household maximizes its utility under a budget constraint where its income is obtained exclusively from firms' profit. In formal terms:

$$\max_{x_i(\omega)} \prod_{i \in I^{\mathfrak{g}}} x_i(\omega)^{\gamma_i}$$

$$\sum_{i \epsilon I^{\mathfrak{g}}} p_i^{\mathfrak{g}}(\omega) x_i(\omega) = \pi(\omega)$$

where $x_i(\omega)$ is the consumption of good i, γ_i is the consumption coefficient of good i ($\sum_{i \in I^{\mathfrak{g}}} \gamma_i = 1, \gamma_i > 0$), and $\pi(\omega)$ is the total of firms' profit in the realization $\omega \in \Omega$. The solution to this problem is:

$$\forall i \in I^{\mathfrak{g}}, \qquad x_i(\omega) = \gamma_i \frac{\pi(\omega)}{p_i^{\mathfrak{g}}(\omega)} \tag{3}$$

The supply (2) and demand (3) functions can be used to obtain the following equilibrium conditions for final-good market:

$$\forall i \in I^{\mathfrak{g}}, \qquad p_i^{\mathfrak{g}}(\omega) \left(\frac{p_i^{\mathfrak{g}}(\omega) - \sum_k \alpha_{ik} p_{ki}^{\mathfrak{m}}}{\vartheta_i} \right)^{\frac{1}{\vartheta_i - 1}} = \gamma_i \pi(\omega) w^{\frac{1}{\vartheta_i - 1}}$$

Let us denote the right side of these equations by $f_i\left(p_i^{\mathfrak{g}}(\omega)\right)$. Since $\vartheta_i > 1$ and $p_i^{\mathfrak{g}}(\omega) > \sum_k \alpha_{ik} p_{ki}^{\mathfrak{m}}$, we have $\frac{\partial f_i}{\partial p_i^{\mathfrak{g}}} > 0$.[63]

Due to the Walras Law, one of these equations is irrelevant so it is necessary to choose a numeraire to solve the system[64]. If the household income is taken as a numeraire (i.e. $\pi(\omega) = \bar{\pi}$), then there is a price $p_i^{\mathfrak{g}*}(\omega, w)$ that solves:

---

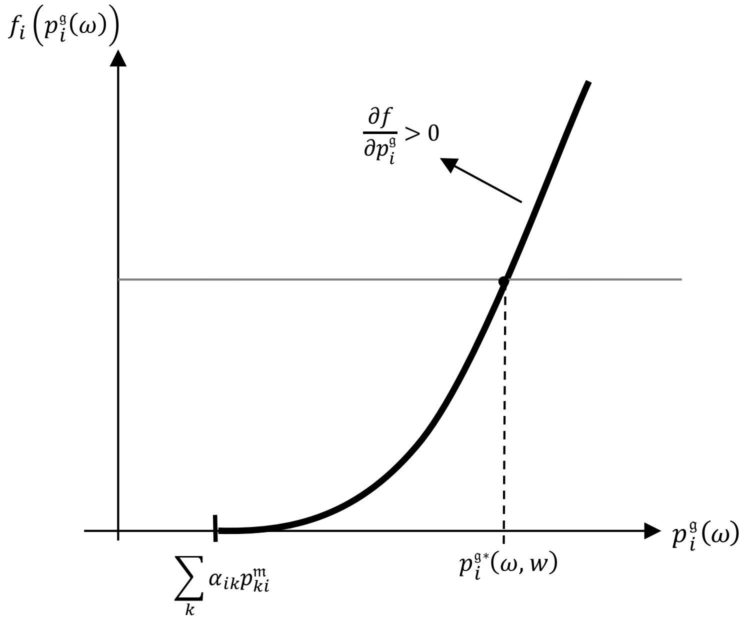
[63] $\frac{\partial f_i}{\partial p_i^{\mathfrak{g}}} = f_i\left(p_i^{\mathfrak{g}}(\omega)\right) . \left(\frac{1}{p_i^{\mathfrak{g}}(\omega)} + \frac{1}{(\vartheta_i - 1)(p_i^{\mathfrak{g}}(\omega) - \sum_k \alpha_{ik} p_{ki}^{\mathfrak{m}})} \right) > 0$

[64] An important characteristic about the transactions of the productive network $R = (V, \Gamma)$ is that the representative household's income is equal to the total sales of final-good firms. This fact is generated by the accumulation of added value in the productive network (the sales of the final-good firms are equal to the added value plus the input costs of these firms. These costs are equal to the sales of their suppliers, which in turn are equal to the added value

$$f_i\left(p_i^{g*}(\omega, w)\right) = \gamma_i \bar{\pi} w^{\frac{1}{\vartheta_i - 1}} \tag{4}$$

The solution to this equation could be seen in the following figure:

Figure. Determination of Price $p_i^{g*}(\omega, w)$



As shown, $\lim_{w \to 0} p_i^{g*}(\omega, w) = \sum_k \alpha_{ik} p_{ki}^m$ and $\lim_{w \to \infty} p_i^{g*}(\omega, w) = \infty$

Equation (4) states an interesting result: If the wage is known, the price of final goods that clears the market will also be known. To find this wage, it is necessary to analyze the labor market. Here, the equilibrium is determined by:

$$\bar{L} = \sum_{j \in J} l_j\left(y_j(\omega)\right) = \sum_{j \in J} l_j\left(\sum_{i \in I^g} \theta_i(\omega) \hat{y}_i(\omega)\right)$$

where $\bar{L}$ is the fixed total labor supply. By equations (1) and (2) this equilibrium can be expressed as:

---

and the input costs of these firms, and so on from supplier to supplier). Because of the budget constraint, representative household's income is also equal to their spending in final goods, so the total excess demand is zero (ed the Walras Law is true). In this sense, it is possible to choose as a numeraire any linear combination of the prices of final goods or the total household income.

$$\bar{L} = \sum_{j \in J} l_j \left(\sum_{i \in I^{\mathfrak{g}}} \theta_i(\omega) \left(\frac{p_i^{\mathfrak{g}}(\omega) - \sum_k \alpha_{ik} p_{ki}^{\mathfrak{m}}}{w \vartheta_i} \right)^{\frac{1}{\vartheta_i - 1}} \right) \tag{5}$$

Suppose this equilibrium is achieved by the prices $p_i^{\mathfrak{g}*}(\omega, w)$ that solve (4). In this sense, the total labor demand (left side of equation (5)) can be represented as a function $g\left(p_i^{\mathfrak{g}*}(\omega, w)\ , w\right)$. Let's examine the properties of this function.

First, if the wage is close to zero, by L'Hopital theorem[65] we have:

$$\lim_{w \to 0} \left(\frac{p_i^{\mathfrak{g}*}(\omega, w) - \sum_k \alpha_{ik} p_{ki}^{\mathfrak{m}}}{w \vartheta_i} \right)^{\frac{1}{\vartheta_i - 1}} = \frac{\gamma_i \bar{\pi}}{\sum_k \alpha_{ik} p_{ki}^{\mathfrak{m}}}$$

thus:

$$\lim_{w \to 0} g\left(p_i^{\mathfrak{g}*}(\omega, w), w\right) = \sum_{j \in J} l_j \left(\frac{\gamma_i \bar{\pi}}{\sum_k \alpha_{ik} p_{ki}^{\mathfrak{m}}} \right) > 0$$

Second, if the wage tends to infinity, applying in the same way L'Hopital theorem we have:

$$\lim_{w \to 0} \left(\frac{p_i^{\mathfrak{g}*}(\omega, w) - \sum_k \alpha_{ik} p_{ki}^{\mathfrak{m}}}{w \vartheta_i} \right)^{\frac{1}{\vartheta_i - 1}} = 0$$

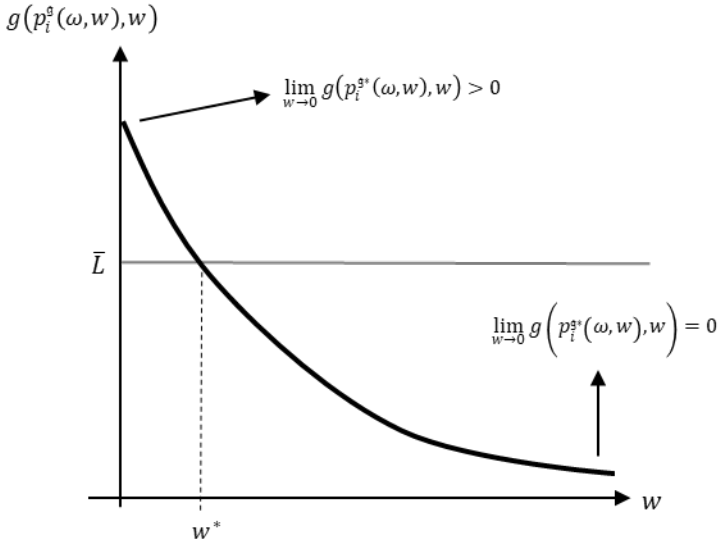
thus:

$$\lim_{w \to 0} g\left(p_i^{\mathfrak{g}*}(\omega, w), w\right) = 0$$

Given these properties, there is a wage w^* that clears the labor market. This fact can be seen in the following figure.

[65] Using implicit differentiation in (4) , we have: $\frac{dp_i^{\mathfrak{g}}}{dw} = \frac{\vartheta_i (\gamma_i \bar{\pi})^{(\vartheta_i - 1)}}{(\vartheta_i - 1) p_i^{\mathfrak{g}\,(\vartheta_i - 2)} (p_i^{\mathfrak{g}} - \sum_k \alpha_{ik} p_{ki}^{\mathfrak{m}}) + p_i^{\mathfrak{g}\,(\vartheta_i - 1)}}$

Figure. Determination of wage w^*



In summary, the equilibrium wage in (5) can be used to determine the prices of final goods by (4). Both variables determine the production of final goods by (2), the final consumption by (3) and, consequently, the production of all firms in the productive network by (1). Therefore, the system $H(x(\omega), y(\omega), p^{\mathrm{g}}(\omega), w(\omega), p^{\mathrm{m}})$ has a solution. This implies that the variables $x(\omega), y(\omega)$ and the prices $p^{\mathrm{g}}(\omega), w(\omega)$ are determined for any realization $\omega \in \Omega$, so the expected value $E\left[\pi_j / p_j^{\mathrm{m}}, p_{-j}^{\mathrm{m}}\right]$ for any firm $j \in J^{\mathrm{m}}$ is well defined.

E.2. The prices of intermediate goods p^{m} belongs to a convex and compact set

It is known that $p^{\mathrm{m}} \in \mathbb{R}_+^N \times \mathbb{R}_+^{N^{\mathrm{m}}}$. This set is closed and convex by definition, however, it is not bounded. This condition is necessary for compactness.

In this sense, it is necessary to demostrate if there is a real number such that the prices are lower than it. As stated in the previous section, Walras Law requires taking a numeraire to solve the system of equilibrium equations. If a price index of the form $I = \sum_k \gamma_i p_i^{\mathrm{g}}(\omega)$ is taken as a numeraire, then:

$$\forall i \in I^{\mathrm{g}}, \qquad p_i^{\mathrm{g}}(\omega) \leq I$$

Using the activity conditions for the final-good firms, we have:

$$\forall i \in I^{\mathrm{g}}, \qquad p_i^{\mathrm{g}}(\omega) > \sum_{k \in b(i)} \alpha_{ik} p_{ki}^{\mathrm{m}}$$

since $0 \leq \alpha_{ik} \leq 1$, then:

$$\forall i \in I^{\mathfrak{g}}, \forall k \in b(i), \qquad p_{ki}^{\mathfrak{m}} < I$$

Likewise, the activity conditions for intermediate goods firms implies:

$$\forall j \in J^{\mathfrak{m}}, \forall k \in \Delta_j^{+}(R), \qquad p_{jk}^{\mathfrak{m}} > \sum_{i \in b(j)} \sum_{\substack{k' \in \\ \Delta_{\bar{j}i}(\omega)}} \alpha_{ji} p_{k'j}^{\mathfrak{m}}$$

Thus:

$$\forall j \in J^{\mathfrak{m}}, \forall k \in \Delta_j^{+}(R), \;\; p_{jk}^{\mathfrak{m}} < I$$

Hence, $p^{\mathfrak{m}}$ belongs to a convex and compact set $A = \left\{p \in \mathbb{R}_+^N \times \mathbb{R}_+^{N^{\mathfrak{m}}} \mid \forall j, k \in J^{\mathfrak{m}}, \;\; p_{jk} < I\right\}$.

E.3. The expected profit $E\left[\pi_j / p_j^{\mathfrak{m}}, p_{-j}^{\mathfrak{m}}\right]$ for any firm $j \in J^{\mathfrak{m}}$ is a continuous function of $p_j^{\mathfrak{m}}$

Before doing this step, let's first consider the probabilistic choice problem when the trade-off function has a linear specification.

$$\begin{aligned}
&\max_{\psi} \; \psi_{jk}\Pi_j + \left(1 - \psi_{jk}\right)\Pi_{\bar{j}} \\
&s.r \\
&f\left(\psi_{jk}\right) \geq \beta \\
&0 \leq \psi_{jk} \leq 1
\end{aligned}$$

where ψ_{jk} is the probability that the buyer assigns to transacting with the regular supplier j; $\Pi_j, \Pi_{\bar{j}}$ are the expected profits obtained from trade with the regular supplier j and the rival supplier $\bar{j}$, respectively, β is the minimum preference value for the trade-off, and $f\left(\psi_{jk}\right)$ is the following linear function:

$$f\left(\psi_{jk}\right) = \left(\Pi_{\bar{j}} - \Pi_j\right) + a\left(1 - \psi_{jk}\right)\left(\phi_{\bar{j}}\varepsilon_{\bar{j}} - \phi_j\varepsilon_j\right), \qquad a > 0$$

Here, it is assumed that $\beta < \left(\Pi_{\bar{j}} - \Pi_j\right) < \beta - a\left(\phi_{\bar{j}}\varepsilon_{\bar{j}} - \phi_j\varepsilon_j\right)$ in order to obtain an internal solution. In this way, the probabilistic choice is determined by:

$$\psi_{jk} = 1 - \frac{\beta - \left(\Pi_{\bar{j}} - \Pi_j\right)}{a\left(\phi_{\bar{j}}\varepsilon_{\bar{j}} - \phi_j\varepsilon_j\right)} \tag{6}$$

Due to Marx-Leontief technology, it is known that $\Pi_{\bar{j}} - \Pi_j = -\alpha_{k,a(j)}\left(p^{\mathfrak{m}}_{jk} - p^{\mathfrak{m}}_{\bar{j}k}\right)$, so the probability function is:

$$\psi_{jk} = 1 - \frac{\beta + \alpha_{k,a(j)}\left(p^{\mathfrak{m}}_{jk} - p^{\mathfrak{m}}_{\bar{j}k}\right),}{a\left(\phi_{\bar{j}}\varepsilon_{\bar{j}} - \phi_j\varepsilon_j\right)} \tag{7}$$

Here, it can be checked that:

$$\begin{aligned} \frac{\partial\psi_{jk}}{\partial p^{\mathfrak{m}}_{jk}} &= \frac{1}{a\left(\phi_{\bar{j}}\varepsilon_{\bar{j}} - \phi_j\varepsilon_j\right)} < 0, \qquad \frac{\partial^2\psi_{jk}}{\partial {p^{\mathfrak{m}}_{jk}}^2} = 0 \\ \frac{\partial\psi_{\bar{j}k}}{\partial p^{\mathfrak{m}}_{jk}} &= \frac{1}{a\left(\phi_{\bar{j}}\varepsilon_{\bar{j}} - \phi_j\varepsilon_j\right)} < 0, \qquad \frac{\partial^2\psi_{\bar{j}k}}{\partial {p^{\mathfrak{m}}_{jk}}^2} = 0 \end{aligned} \tag{8}$$

In other words, the probability functions ψ_{jk} y $\psi_{\bar{j}k}$ are decreasing and concave continuous functions in the prices $p^{\mathfrak{m}}_{jk}$ y $p^{\mathfrak{m}}_{\bar{j}k}$, respectively. These results are important points in the demonstration.

Now we can return to step 4. As it will be recalled, the expected profit of intermediate good firms is:

$$E\left[\pi_j / p^{\mathfrak{m}}_j, p^{\mathfrak{m}}_{-j}\right] = \sum_{\omega\in\Omega} \Psi\left(\omega / p^{\mathfrak{m}}_j, p^{\mathfrak{m}}_{-j}\right)\ \pi_j\left(p^{\mathfrak{m}}_j, p^{\mathfrak{m}}_{-j}, \omega\right)$$

On the one side, the probability $\Psi\left(\omega / p^{\mathfrak{m}}_j, p^{\mathfrak{m}}_{-j}\right)$ is equal to the multiplication of the probabilities $\psi_{jk}\left(p^{\mathfrak{m}}_{jk}, p^{\mathfrak{m}}_{\bar{j}k}\right)$, which have the continuous form (7). On the other side, the profit $\pi_j\left(p^{\mathfrak{m}}_j, p^{\mathfrak{m}}_{-j}, \omega\right)$ for each realization $\omega \in \Omega$ is:

$$\pi_j\left(p^{\mathfrak{m}}_j, p^{\mathfrak{m}}_{-j}, \omega\right) = \sum_{k\in\Delta^+_j(\omega)} p^{\mathfrak{m}}_{jk}\, z_{k,a(j)}\left(y_k, p^{\mathfrak{m}}_{jk}, p^{\mathfrak{m}}_{-j}, w(\omega)\right) - c_j\left(y_j, p^{\mathfrak{m}}_{-j}, w(\omega)\right)$$

Here, the conditional demand $z_{k,a(j)}$ and the minimum cost function c_j are determined from the cost minimization problem with a Marx-Leontief technology. Formally, this problem for firm j has the following formulation:

$$\min_{z_{ji}} \sum_{i\in b(j)} \sum_{\substack{k\in \\ \Delta^-_{ji}(\omega)}} p^{\mathfrak{m}}_{kj}\, z_{ji} + w\, l_j$$

$$\min\left\{\frac{z_{j1}}{\alpha_{j1}},\dots,\frac{z_{jn}}{\alpha_{jn}},\frac{l_j}{\alpha_{jl}}\right\}=y_j$$

Solving this problem, we obtain [66]:

$$l_j=\alpha_{jl}w \qquad z_{ji}=\alpha_{ji}y_j, \qquad \forall i\in b(j)$$

$$c_j=y_j\left(\sum_{i\in b(j)}\sum_{\substack{k\in\\ \Delta_{ji}^{-}(\omega)}}\alpha_{ji}p_{kj}^{\mathfrak{m}}+\alpha_{jl}w\right), \qquad y_j=\sum_{\substack{k\in\\ \Delta_j^{+}(\omega)}}z_{k,a(j)}$$

thus, the profit in realization $\omega\in\Omega$ is:

$$\pi_j\left(p_j^{\mathfrak{m}},p_{-j}^{\mathfrak{m}},\omega\right)=\sum_{k\in\Delta_j^{+}(\omega)}\alpha_{k,a(j)}p_{jk}^{\mathfrak{m}}\,y_k-y_j\left(\sum_{i\in b(j)}\sum_{\substack{k\in\\ \Delta_{ji}^{-}(\omega)}}\alpha_{ji}p_{kj}^{\mathfrak{m}}+w\alpha_{jl}\right) \tag{9}$$

As noted, this function is linear and continuous in $p_j^{\mathfrak{m}}$. Hence, because of the fact that $\Psi\left(\omega/p_j^{\mathfrak{m}},p_{-j}^{\mathfrak{m}}\right),\pi_j\left(p_j^{\mathfrak{m}},p_{-j}^{\mathfrak{m}},\omega\right)$ are continuous functions, it can be concluded that the expected profit $E\left[\pi_j/p_j^{\mathfrak{m}},p_{-j}^{\mathfrak{m}}\right]$ is also a continuous function.

E.4. The expected profit $E\left[\pi_j/p_j^{\mathfrak{m}},p_{-j}^{\mathfrak{m}}\right]$ for a firm $j\in J^{\mathfrak{m}}$ is a concave function in $p_j^{\mathfrak{m}}$

To do this step, it is necessary to make two observations about the expected profit $E\left[\pi_j/p_j^{\mathfrak{m}},p_{-j}^{\mathfrak{m}}\right]$. First, the probability function $\Psi\left(\omega/p_j^{\mathfrak{m}},p_{-j}^{\mathfrak{m}}\right)$ can be rewritten as follows:

$$\Psi\left(\omega/p_j^{\mathfrak{m}},p_{-j}^{\mathfrak{m}}\right)=\Psi_{-j}\left(\omega/p_{-j}^{\mathfrak{m}}\right)\Psi_j\left(\omega/p_j^{\mathfrak{m}}\right)$$

where Ψ_{-j} is the multiplication of all transaction probabilities in the productive network, except those transactions made by the firm j with its buyers $\Delta_j^{+}(\omega)$; and Ψ_j is the multiplication of the probabilities for those transactions made by the firm j with its buyers $\Delta_j^{+}(\omega)$. Formally:

[66] Recall that $\Delta_j^{+}(\omega)$ and $\Delta_{ji}^{-}(\omega)$ are operators that show the set of buyers and suppliers of the firm j in realization $\omega\in\Omega$, respectively.

.

$$\Psi_{-j}(\omega/p^{\mathfrak{m}}_{-j}) = \prod_{r\neq j} \prod_{\substack{s\in \\ \Delta_r^+(\omega)}} (\psi_{rs})^{\gamma_{rs}(\omega)}$$

$$\Psi_j(\omega/p^{\mathfrak{m}}_j) = \prod_{\substack{k\in \\ \Delta_j^+(\omega)}} (\psi_{jk})^{\gamma_{jk}(\omega)}$$

where $\gamma_{rs}(\omega) = 1 \iff (r,s) \in \hat{\Gamma}(\omega)$.

Second, due to the linear form (9), the set of realizations Ω can be reduced to a set of smaller size $\hat{\Omega}$ containing those realizations in which suppliers trade exclusively with only one buyer. Namely:

$$\hat{\Omega} = \left\{\omega \in \Omega \mid \forall j \in J \ \left|\Delta_j^+(\omega)\right| = 1\right\}$$

Under these considerations, the expected profit $E\left[\pi_j/p^{\mathfrak{m}}_j, p^{\mathfrak{m}}_{-j}\right]$ can be calculated as follows:

$$E\left[\pi_j/p^{\mathfrak{m}}_j, p^{\mathfrak{m}}_{-j}\right] = \sum_{\omega\in\hat{\Omega}} \Psi_{-j}(\omega/p^{\mathfrak{m}}_{-j}) \sum_{\substack{k\in \\ \Delta_j^+(R)}} \alpha_{k,a(j)}\, y_k\, \psi_{jk}(p^{\mathfrak{m}}_{jk}, p^{\mathfrak{m}}_{\bar{j}k}) \left(p^{\mathfrak{m}}_{jk} - \sum_{i\in b(j)} \sum_{\substack{k\in \\ \Delta_{ji}^-(\omega)}} \alpha_{ji} p^{\mathfrak{m}}_{kj} - \alpha_{jl} w \right)$$

Here, the terms $\Psi_{-j}, \left\{y_k \mid k \in \Delta_j^+(R)\right\}, \left\{p^{\mathfrak{m}}_{kj} \mid i \in b(j), k \in \Delta_{ji}^-(\omega)\right\}$ do not depend on $p^{\mathfrak{m}}_j$, so they remain constant for decisions made by the firm j. In this sense, the concavity of the expected profit $E\left[\pi_j/p^{\mathfrak{m}}_j, p^{\mathfrak{m}}_{-j}\right]$ with respect to $p^{\mathfrak{m}}_j$ is given by the following term:

$$h(p^{\mathfrak{m}}_{jk}) = \psi_{jk}(p^{\mathfrak{m}}_{jk}, p^{\mathfrak{m}}_{\bar{j}k}) \left(p^{\mathfrak{m}}_{jk} - \sum_{i\in b(j)} \sum_{\substack{k\in \\ \Delta_{ji}^-(\omega)}} \alpha_{ji} p^{\mathfrak{m}}_{kj} - w\alpha_{jl} \right)$$

The second derivative of this term with respect to $p^{\mathfrak{m}}_{jk}$ is:

$$\frac{\partial^2 h}{\partial {p^{\mathfrak{m}}_{jk}}^2} = \frac{\partial^2 \psi^*_{jk}}{\partial {p^{\mathfrak{m}}_{jk}}^2} p^{\mathfrak{m}}_{jk} + 2\frac{\partial \psi^*_{jk}}{\partial p^{\mathfrak{m}}_{jk}}$$

by (8) we have:

$$\forall k \in J, \forall i \in b(k), \forall j \in c(k,i), \qquad \frac{\partial^2 h}{\partial {p^{\mathfrak{m}}_{jk}}^2} < 0$$

Hence, $h(p_{jk}^{\mathfrak{m}})$ is a concave function with respect to $p_{jk}^{\mathfrak{m}}$. Since any linear combination of concave functions is also concave, then it is proved that $E\left[\pi_j / p_j^{\mathfrak{m}}, p_{-j}^{\mathfrak{m}}\right]$ is a concave function with respect to $p_j^{\mathfrak{m}}$.

E.5. There is a price matrix $\boldsymbol{p}^{\mathfrak{m}*}$ that solves $\boldsymbol{p}_j^{\mathfrak{m}*} = \boldsymbol{F}_j\left(\boldsymbol{p}_{-j}^{\mathfrak{m}*}, \boldsymbol{W}, \boldsymbol{Y}_{-j}\right), \forall \boldsymbol{j} \in \boldsymbol{J}^{\mathfrak{m}}$.

This step uses the results obtained in the previous steps. The Step 1 proves that there is a set of variables $x(\omega), y(\omega), p^{\mathfrak{g}}(\omega), w(\omega)$ such that $H(x(\omega), y(\omega), p^{\mathfrak{g}}(\omega), w(\omega), p^{\mathfrak{m}}) = 0 \ \forall \omega \in \Omega$, so the expected value $E\left[\pi_j / p_j^{\mathfrak{m}}, p_{-j}^{\mathfrak{m}}\right]$ is well defined. The Step 2 proves that $p^{\mathfrak{m}} \in A$, where A is a compact and convex set. Finally, steps 3 and 4 prove that the expected profit $E\left[\pi_j / p_j^{\mathfrak{m}}, p_{-j}^{\mathfrak{m}}\right]$ is a continuous and concave function with respect to $p_j^{\mathfrak{m}}$.

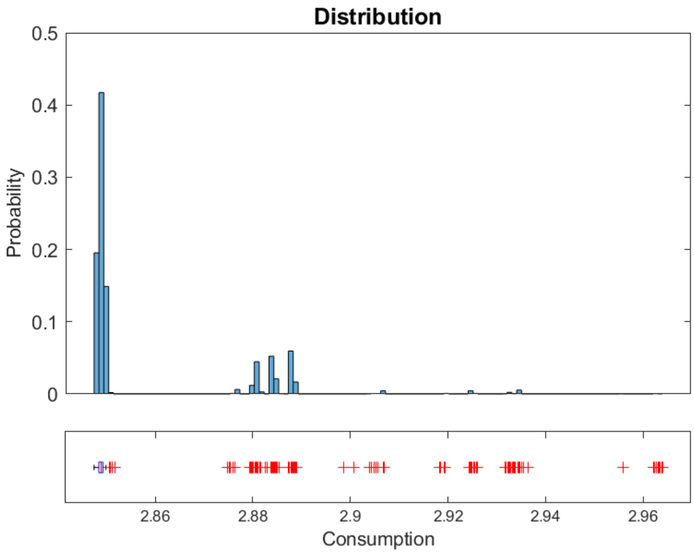
Consequently, using the Nash equilibrium theorem for continuous strategies, there is a price matrix $p^{\mathfrak{m}*}$ that simultaneously maximizes the expected profit of firms. That is, each firm $j \in J^{\mathfrak{m}}$ sets a price vector $p_j^{\mathfrak{m}*}$ that maximizes its profits, assuming that the rest of the firms in the productive network do the same for prices $p_{-j}^{\mathfrak{m}*}$. Formally:

$$\exists p^{\mathfrak{m}*} \in A \mid \forall j \in J^{\mathfrak{m}}, \qquad E\left[\pi_j / p_j^{\mathfrak{m}*}, p_{-j}^{\mathfrak{m}*}\right] > E\left[\pi_j / p_j^{\mathfrak{m}}, p_{-j}^{\mathfrak{m}*}\right], \forall p_j^{\mathfrak{m}} \in A_j$$

Hence, the price matrix $p^{\mathfrak{m}*}$ solves the reaction functions $p_j^{\mathfrak{m}*} = F_j\left(p_{-j}^{\mathfrak{m}*}, W, Y_{-j}\right)$.

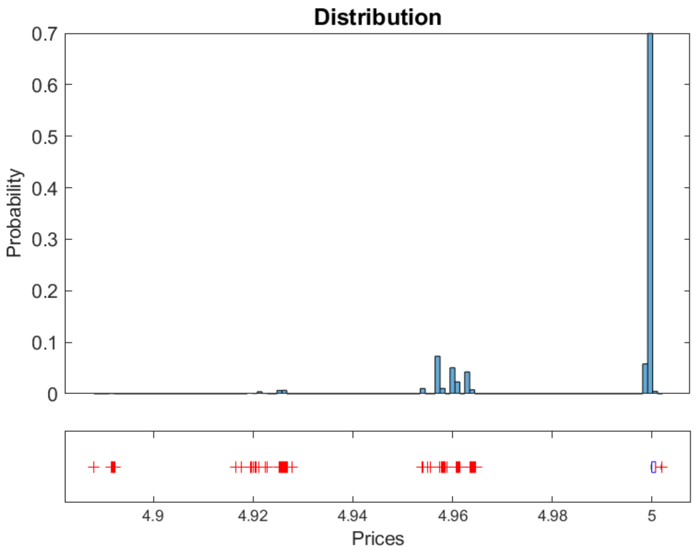
F. Characterization of General Market Equilibrium on base scenario

Figure. General Market Equilibrium. Final consumption price distribution.



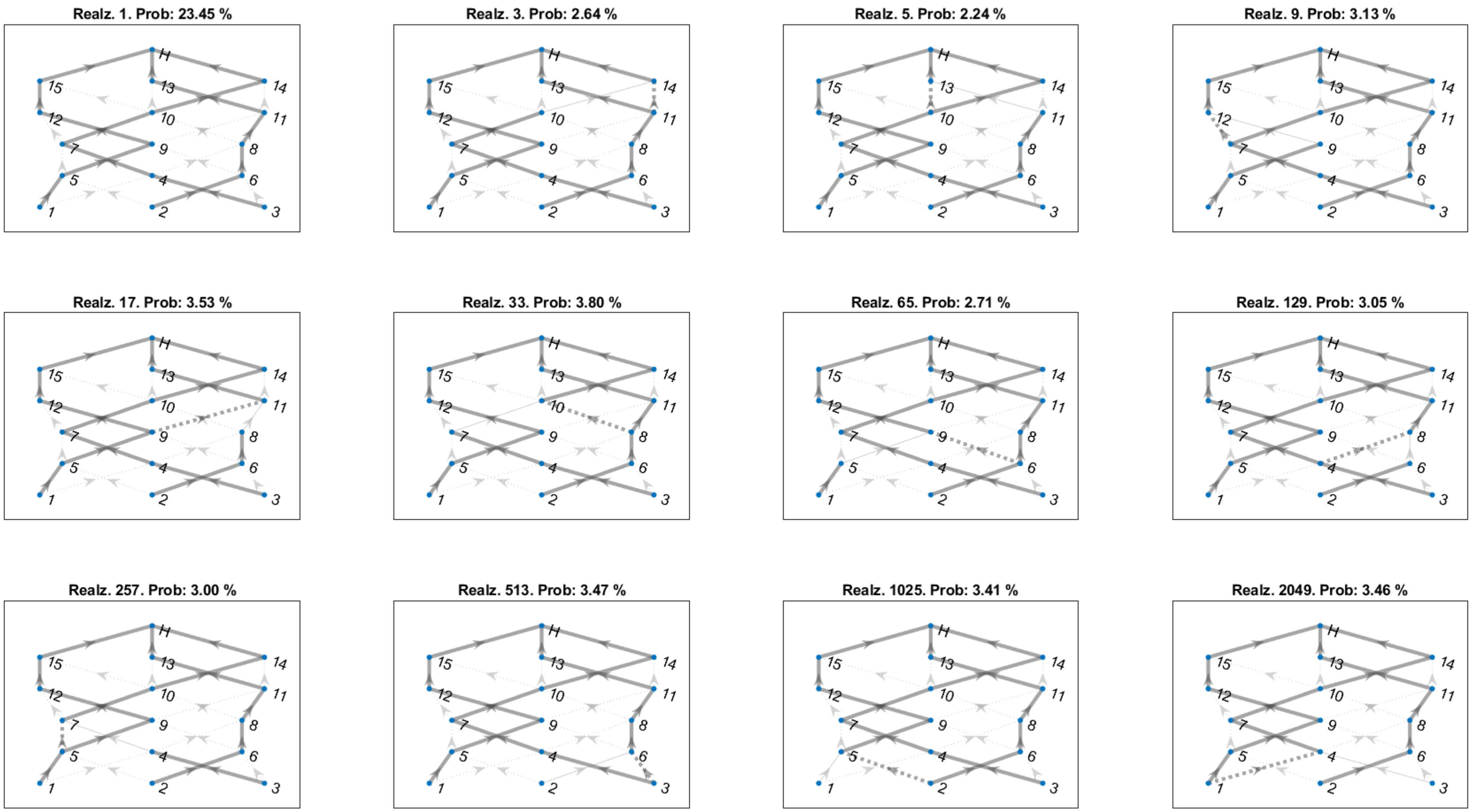
Note. The graphic above shows the distribution of final consumption prices over all the realizations of the productive network. On the other hand, the graph below shows the price dispersion in a straight line.

Figure. General Market Equilibrium. Final consumption distribution



Note. The graphic above shows the distribution of final consumption over all the realizations of the productive network. On the other hand, the graph below shows the final consumption dispersion in a straight line.

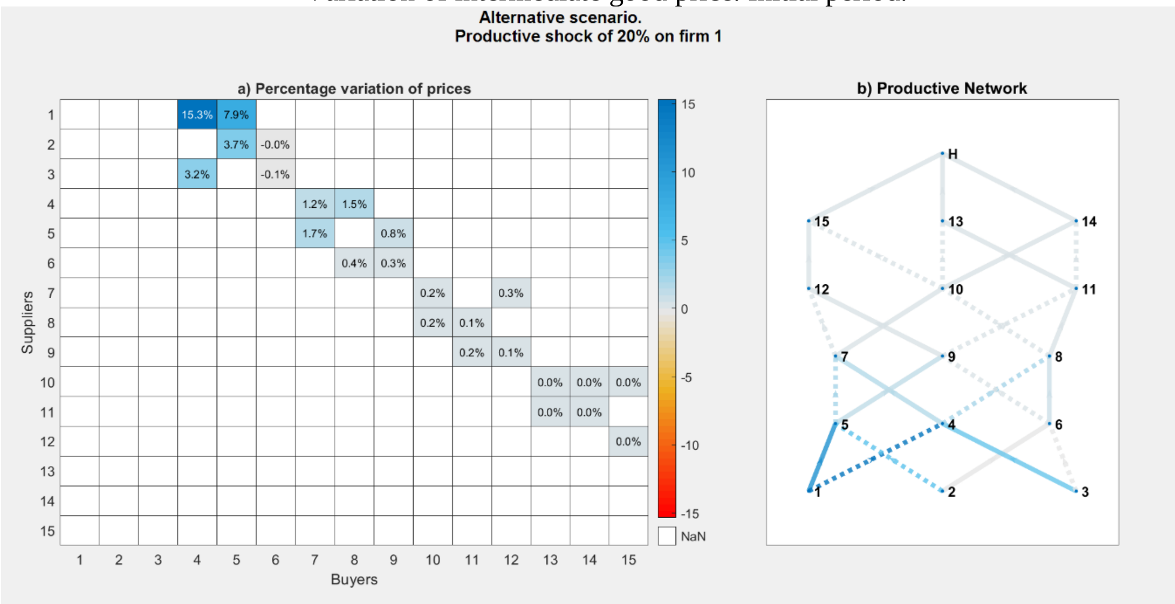
Figure. General Market Equilibrium. Most likely realizations of productive network.

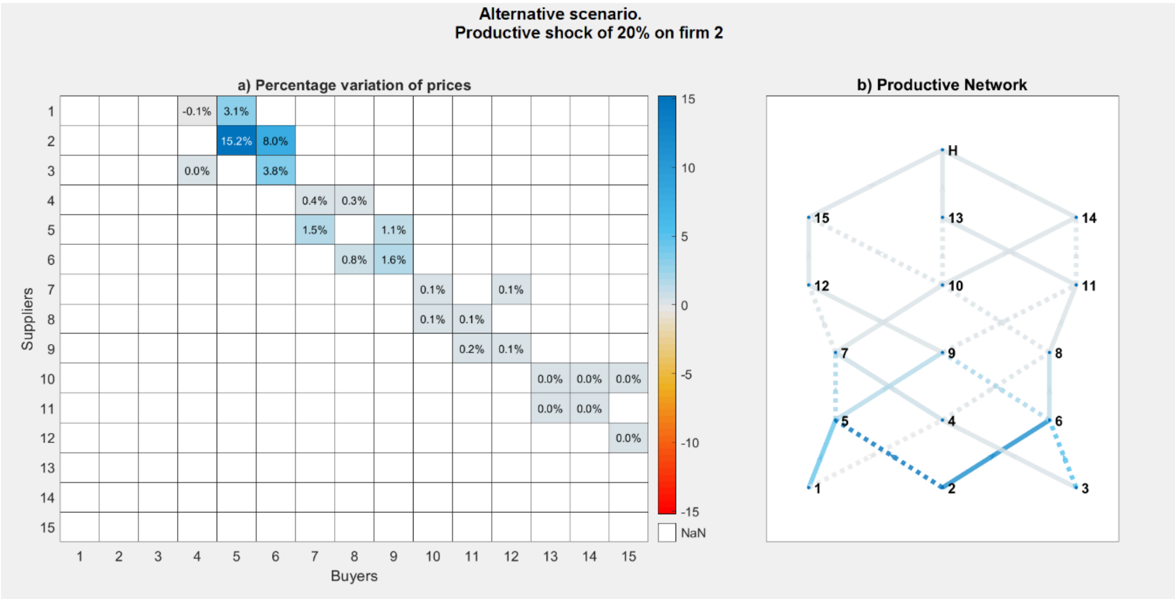


G. Simulation of General Market Equilibrium on alternative scenarios

G.1. Intermediate good price

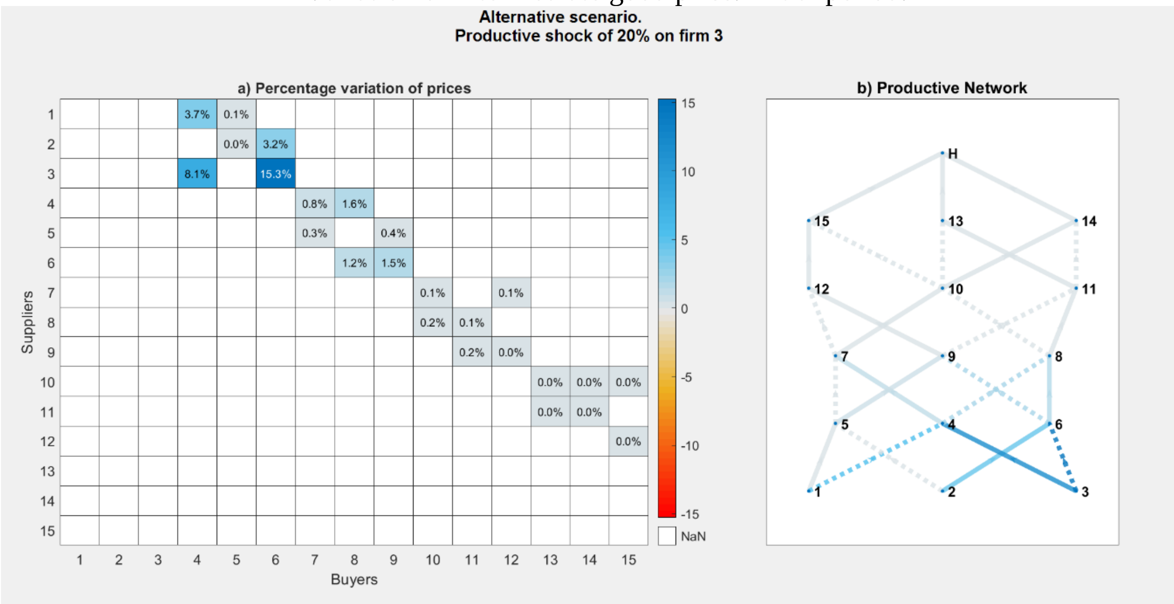
Figures. General Market Equilibrium on alternative scenarios
Variation of intermediate good price. Initial period.

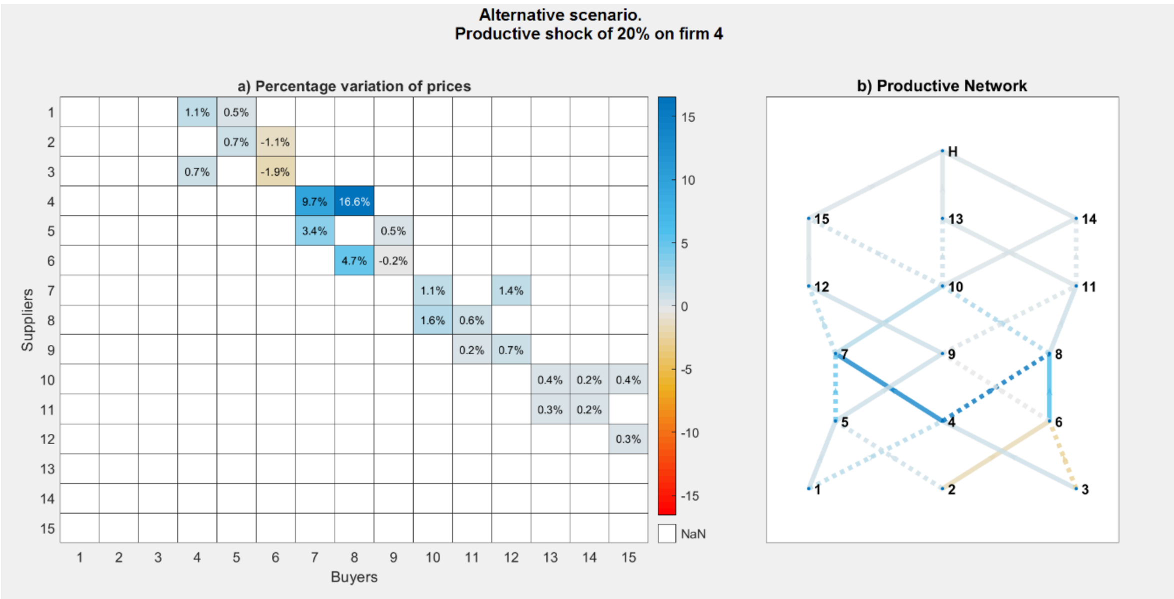




Note. Table a) shows the percentage variation in the selling prices set by the suppliers for their potential buyers. Suppliers are arranged by rows, while buyers are arranged by columns. The red-blue gradation denotes the sign and magnitude of the price variation. If red, then prices decrease; the higher the color red's intensity, the larger the price reduction. On the contrary, if blue, then prices rise; the higher the color blue's intensity, the larger the increase in prices. Figure b) replicates this color grading on the edges of the production network.

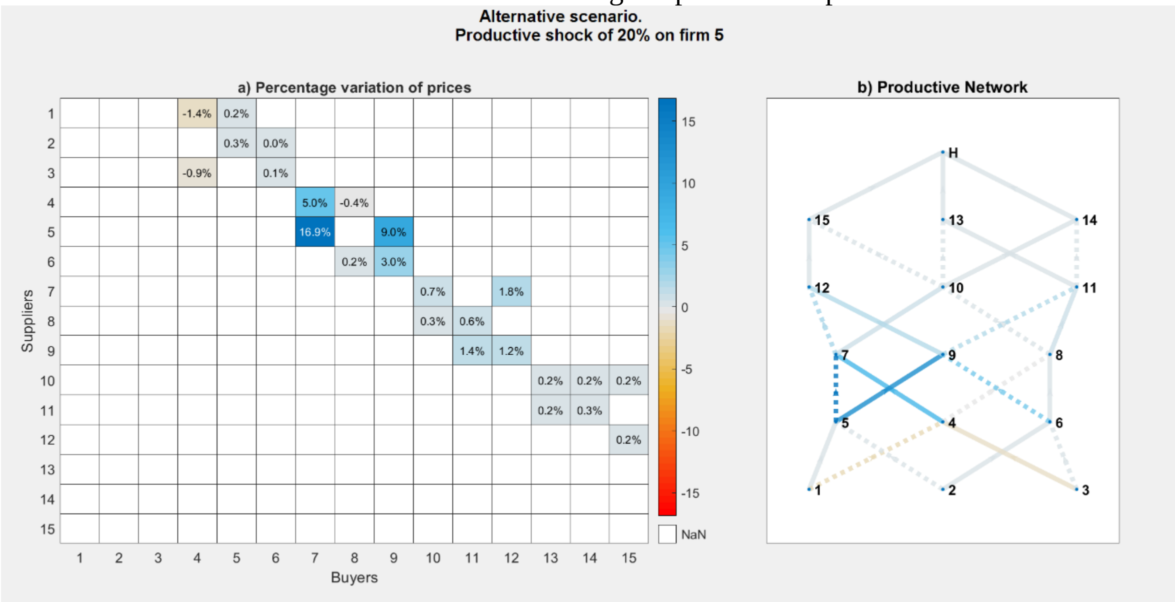
Figures. General Market Equilibrium on alternative scenarios
Variation of intermediate good price. Initial period.

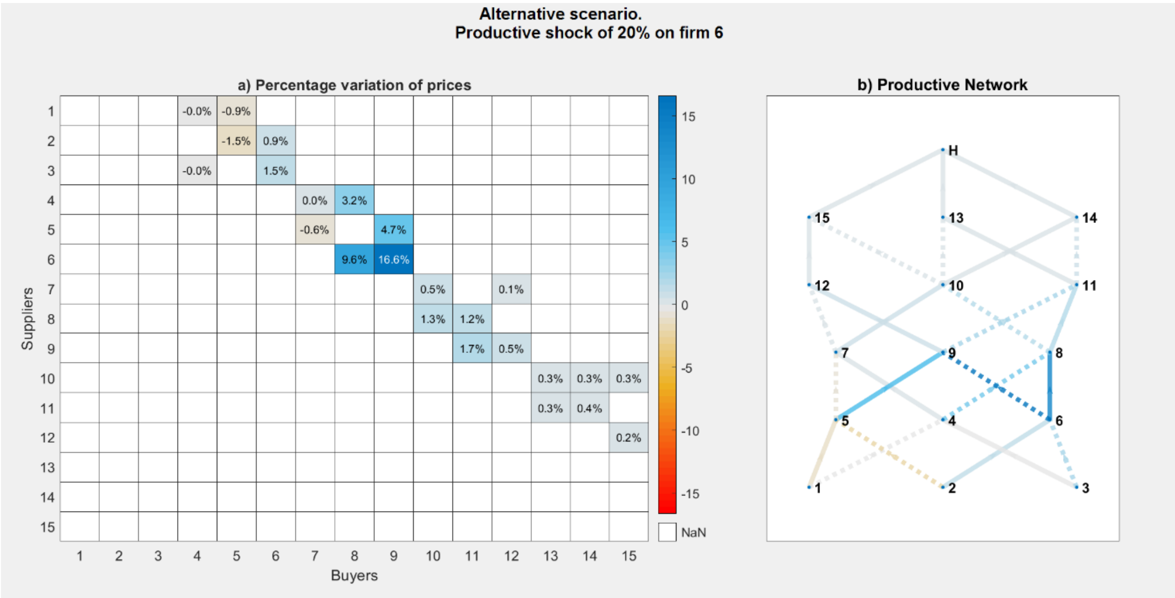




Note. Table a) shows the percentage variation in the selling prices set by the suppliers for their potential buyers. Suppliers are arranged by rows, while buyers are arranged by columns. The red-blue gradation denotes the sign and magnitude of the price variation. If red, then prices decrease; the higher the color red's intensity, the larger the price reduction. On the contrary, if blue, then prices rise; the higher the color blue's intensity, the larger the increase in prices. Figure b) replicates this color grading on the edges of the production network.

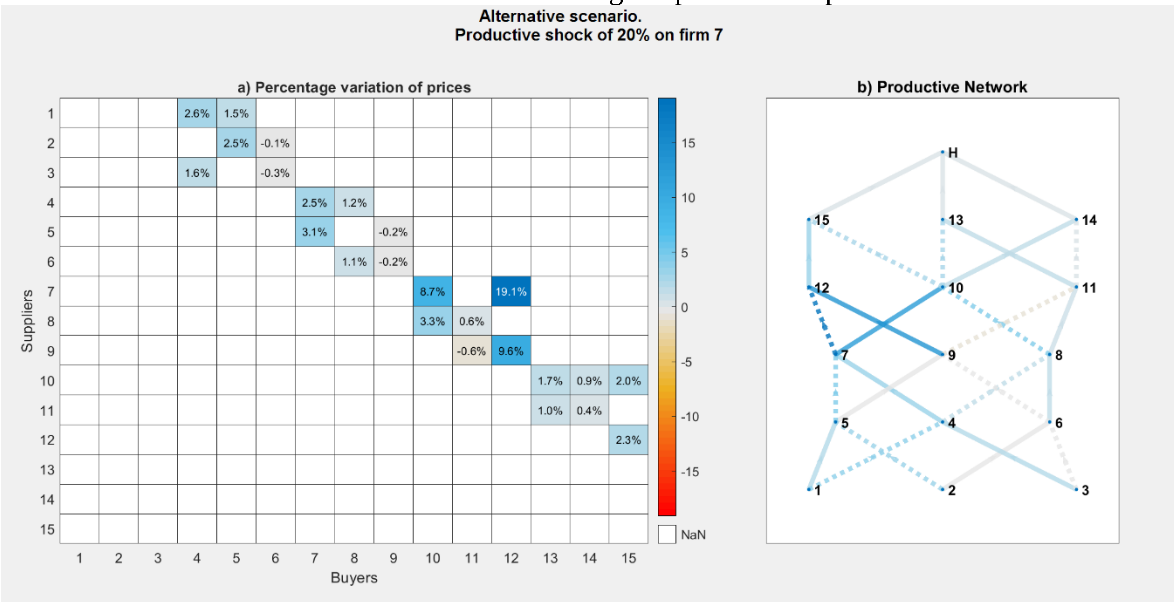
Figures. General Market Equilibrium on alternative scenarios
Variation of intermediate good price. Initial period.

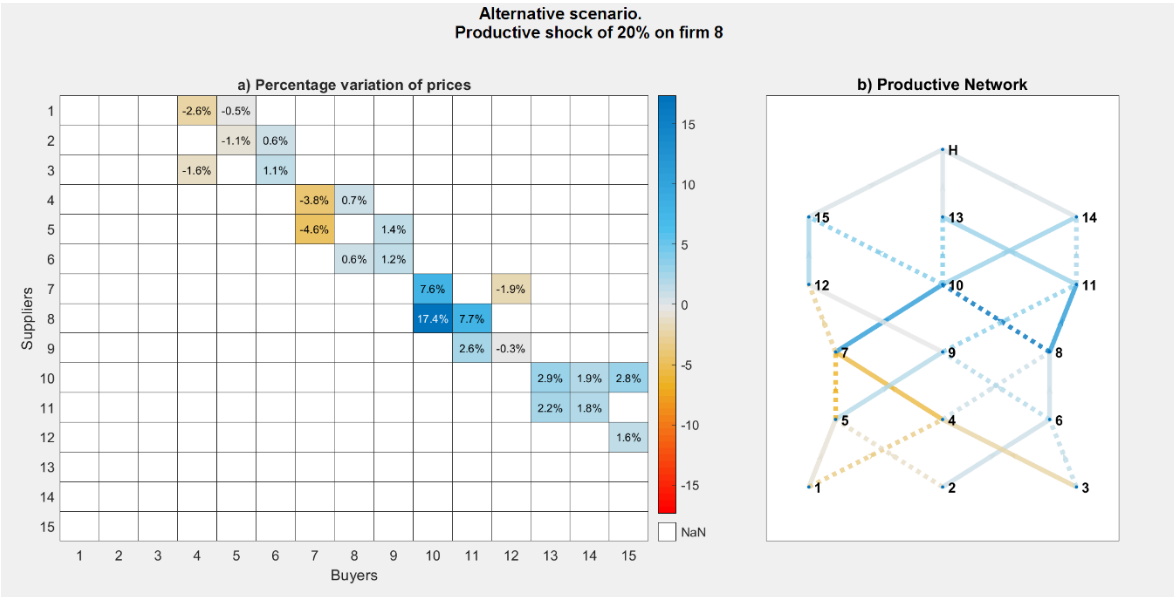




Note. Table a) shows the percentage variation in the selling prices set by the suppliers for their potential buyers. Suppliers are arranged by rows, while buyers are arranged by columns. The red-blue gradation denotes the sign and magnitude of the price variation. If red, then prices decrease; the higher the color red's intensity, the larger the price reduction. On the contrary, if blue, then prices rise; the higher the color blue's intensity, the larger the increase in prices. Figure b) replicates this color grading on the edges of the production network.

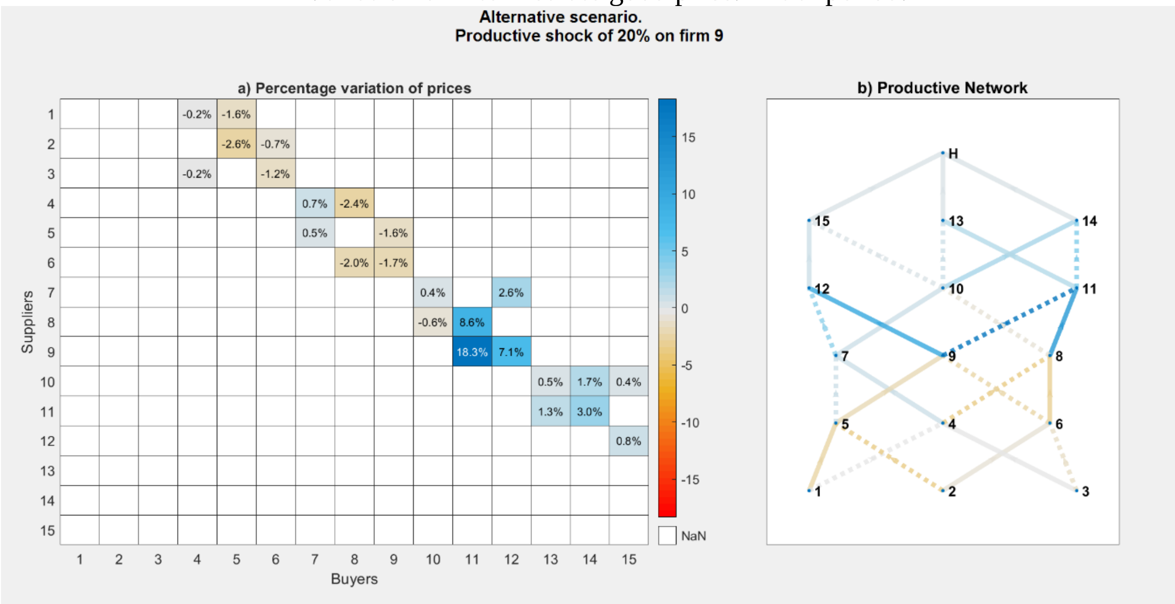
Figures. General Market Equilibrium on alternative scenarios
Variation of intermediate good price. Initial period.

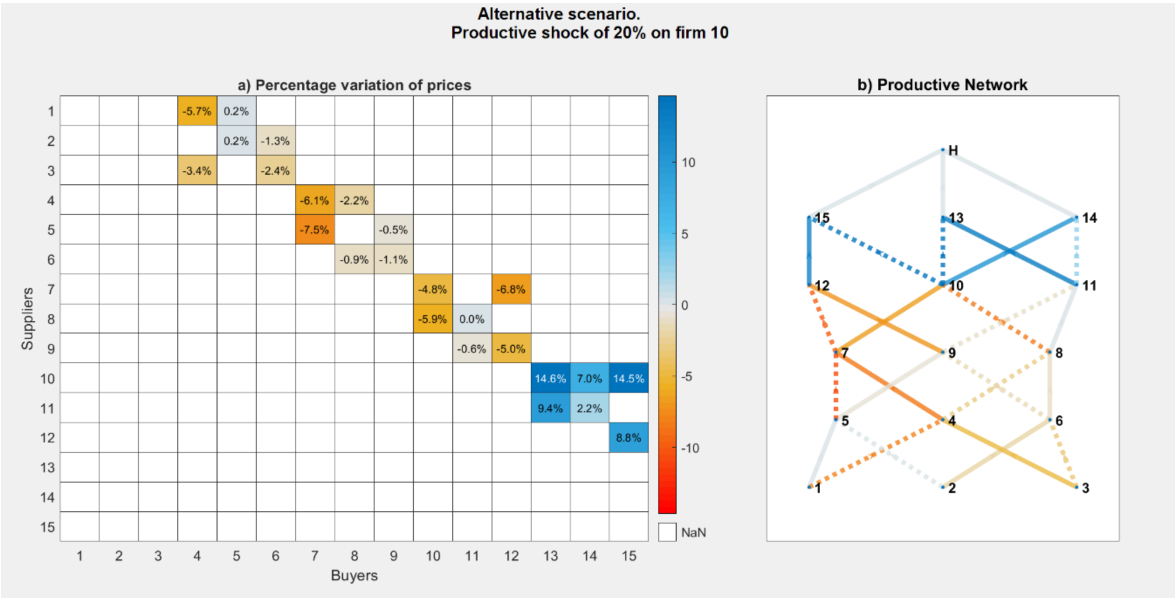




Note. Table a) shows the percentage variation in the selling prices set by the suppliers for their potential buyers. Suppliers are arranged by rows, while buyers are arranged by columns. The red-blue gradation denotes the sign and magnitude of the price variation. If red, then prices decrease; the higher the color red's intensity, the larger the price reduction. On the contrary, if blue, then prices rise; the higher the color blue's intensity, the larger the increase in prices. Figure b) replicates this color grading on the edges of the production network.

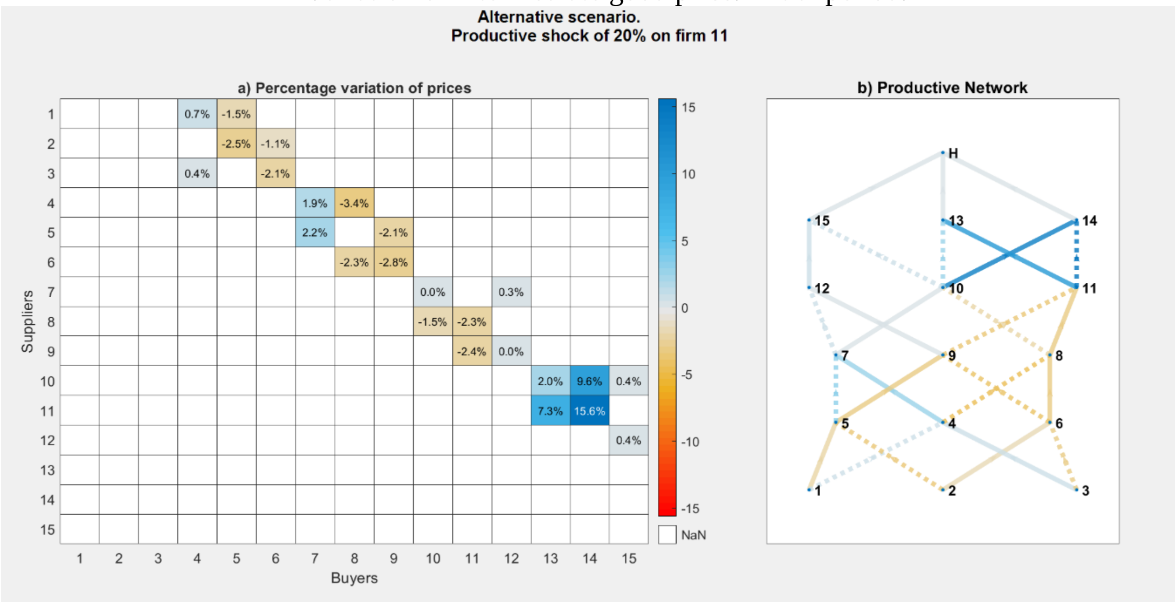
Figures. General Market Equilibrium on alternative scenarios
Variation of intermediate good price. Initial period.

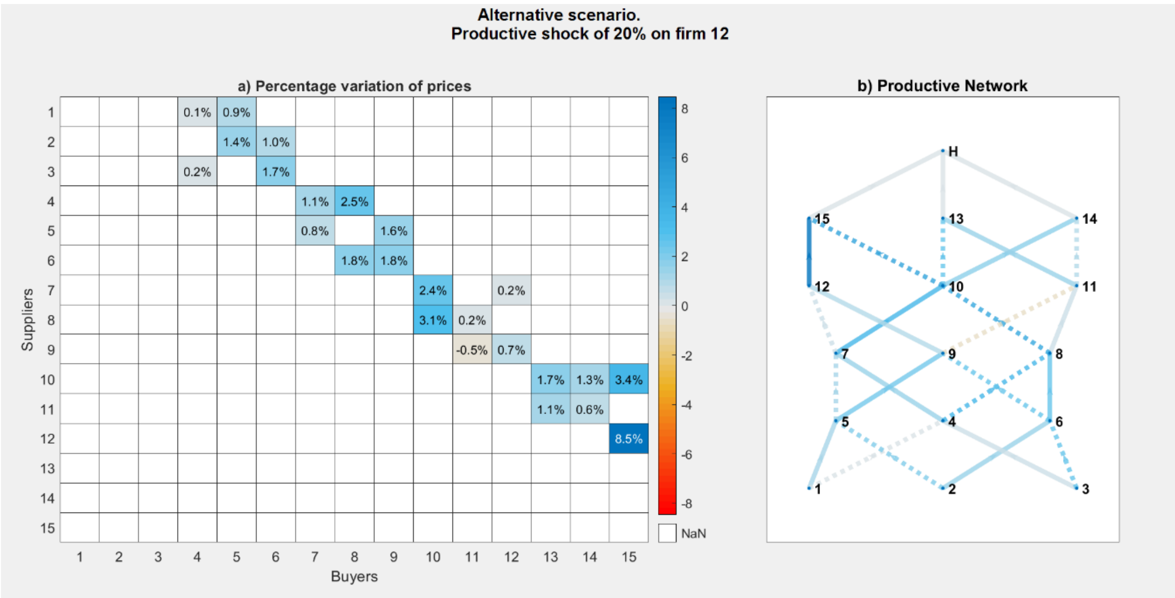




Note. Table a) shows the percentage variation in the selling prices set by the suppliers for their potential buyers. Suppliers are arranged by rows, while buyers are arranged by columns. The red-blue gradation denotes the sign and magnitude of the price variation. If red, then prices decrease; the higher the color red's intensity, the larger the price reduction. On the contrary, if blue, then prices rise; the higher the color blue's intensity, the larger the increase in prices. Figure b) replicates this color grading on the edges of the production network.

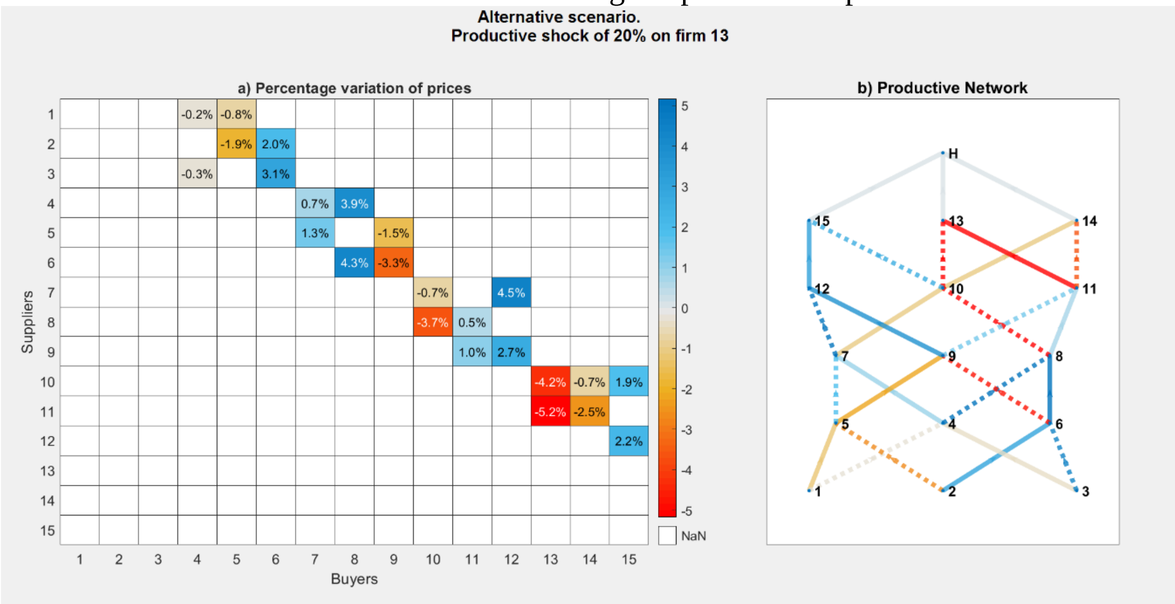
Figures. General Market Equilibrium on alternative scenarios
Variation of intermediate good price. Initial period.

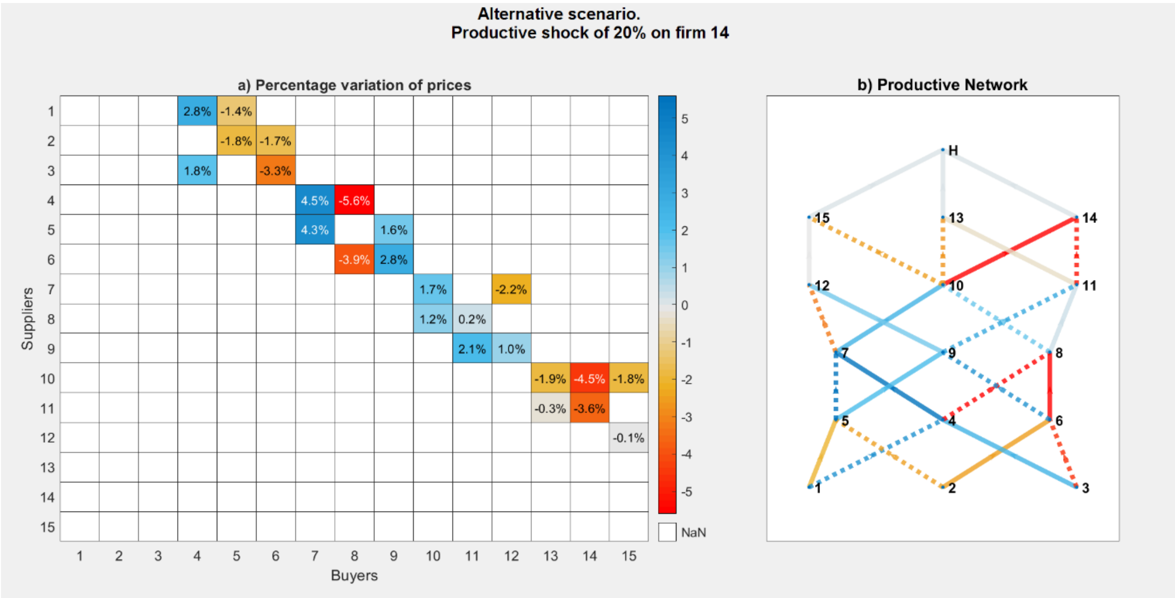




Note. Table a) shows the percentage variation in the selling prices set by the suppliers for their potential buyers. Suppliers are arranged by rows, while buyers are arranged by columns. The red-blue gradation denotes the sign and magnitude of the price variation. If red, then prices decrease; the higher the color red's intensity, the larger the price reduction. On the contrary, if blue, then prices rise; the higher the color blue's intensity, the larger the increase in prices. Figure b) replicates this color grading on the edges of the production network.

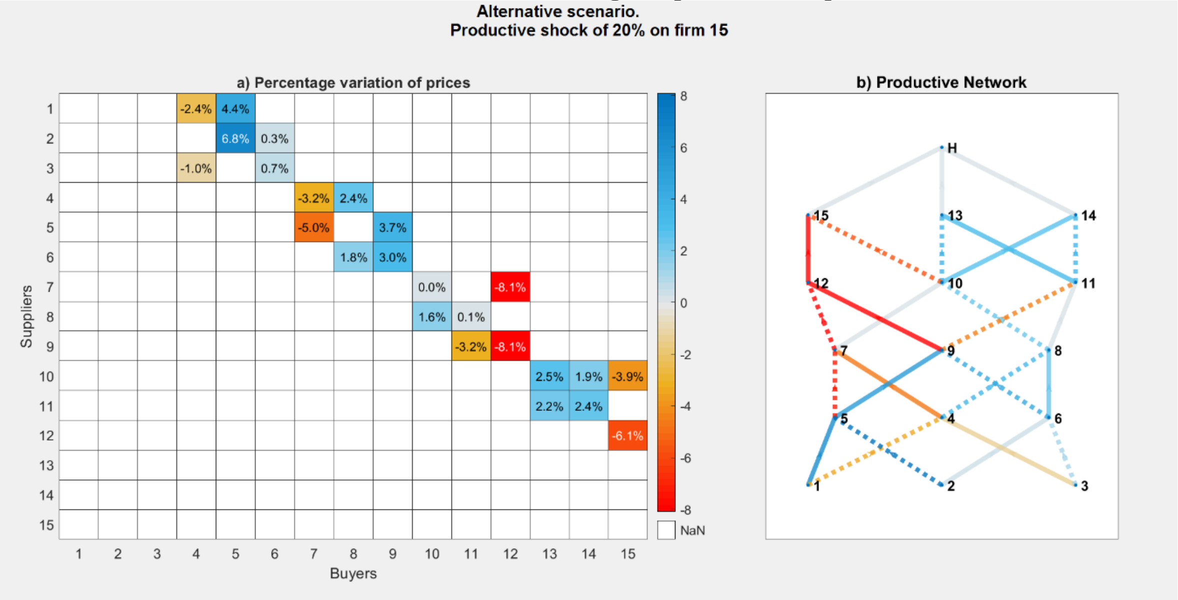
Figures. General Market Equilibrium on alternative scenarios
Variation of intermediate good price. Initial period.





Note. Table a) shows the percentage variation in the selling prices set by the suppliers for their potential buyers. Suppliers are arranged by rows, while buyers are arranged by columns. The red-blue gradation denotes the sign and magnitude of the price variation. If red, then prices decrease; the higher the color red's intensity, the larger the price reduction. On the contrary, if blue, then prices rise; the higher the color blue's intensity, the larger the increase in prices. Figure b) replicates this color grading on the edges of the production network.

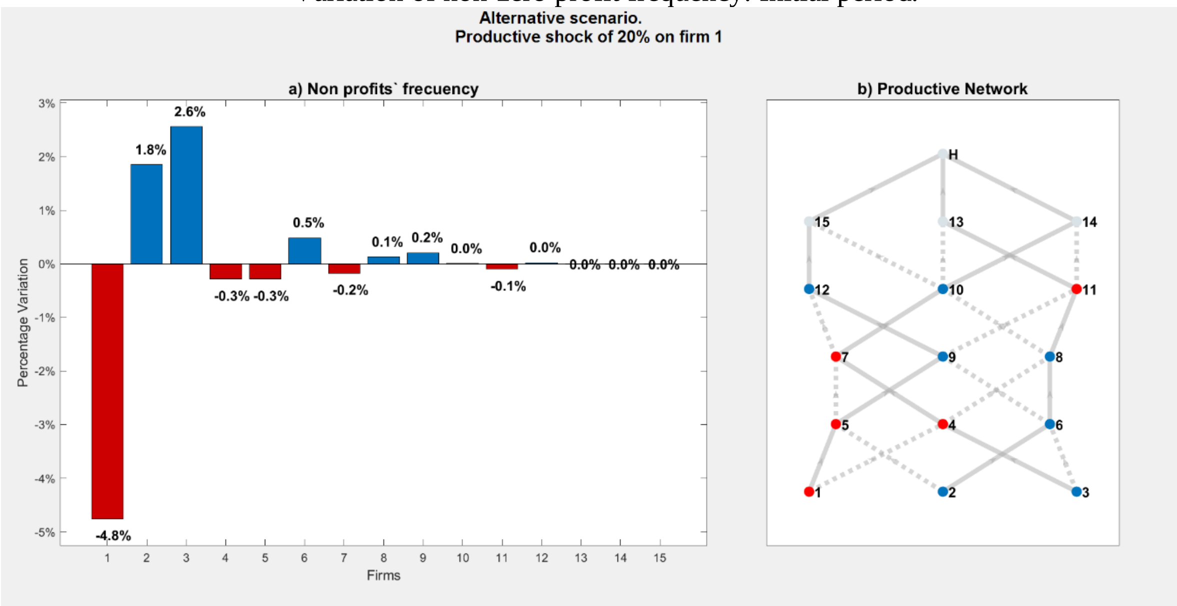
Figures. General Market Equilibrium on alternative scenarios
Variation of intermediate good price. Initial period.

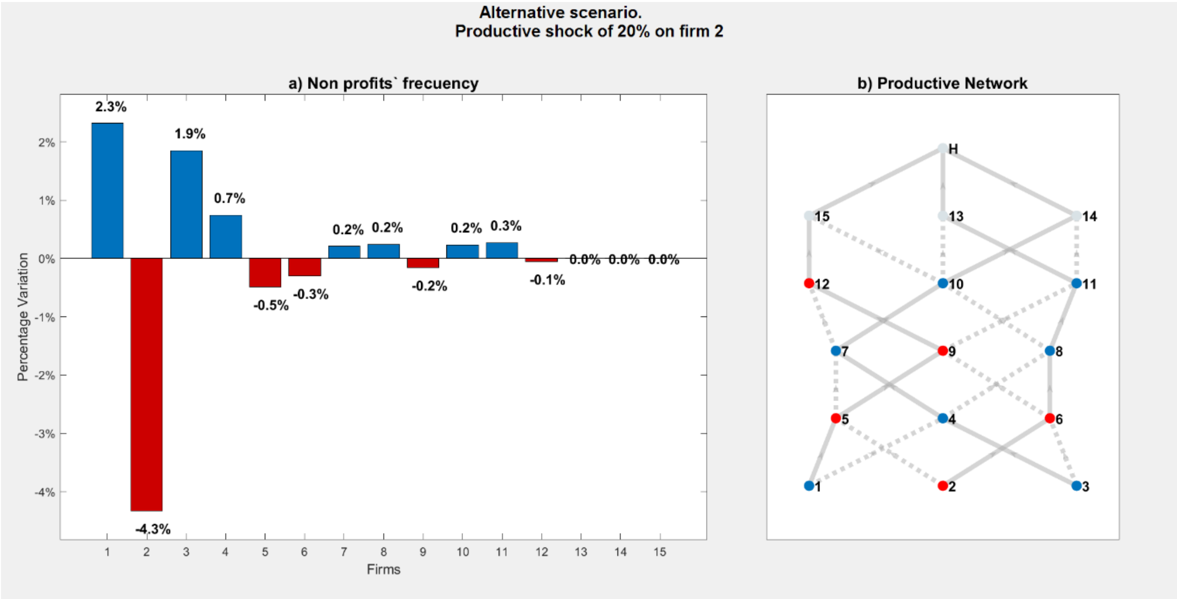


Note. Table a) shows the percentage variation in the selling prices set by the suppliers for their potential buyers. Suppliers are arranged by rows, while buyers are arranged by columns. The red-blue gradation denotes the sign and magnitude of the price variation. If red, then prices decrease; the higher the color red's intensity, the larger the price reduction. On the contrary, if blue, then prices rise; the higher the color blue's intensity, the larger the increase in prices. Figure b) replicates this color grading on the edges of the production network.

G.2. Non-zero profits

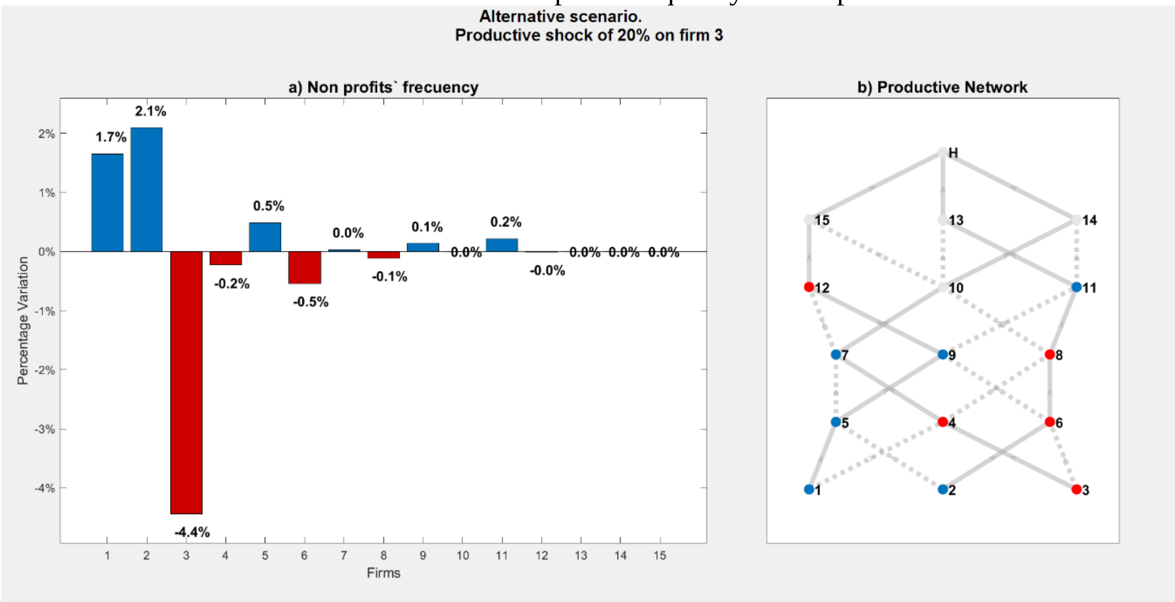
Figures. General Market Equilibrium on alternative scenarios
Variation of non-zero profit frequency. Initial period.

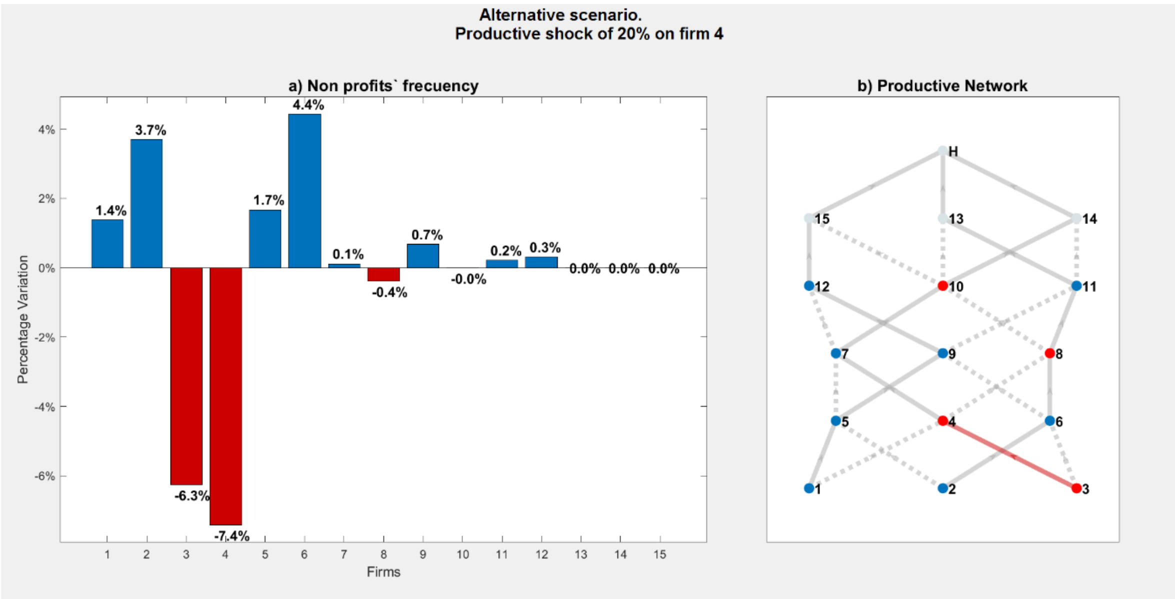




Note. Figure a) is a bar diagram that shows the percentage variation in the frequency with which firms earn non-negative profits. This frequency is computed based on the empirical distribution function of the GME before and after the productivity shock. The red-blue color gradation indicates the sign of the variation. The color red indicates a reduction in the frequency, while a blue coloration denotes an increase in the frequency. Figure b) replicates this color grading on the nodes in the production network and outlines a path with the nodes that exhibit the largest negative variation.

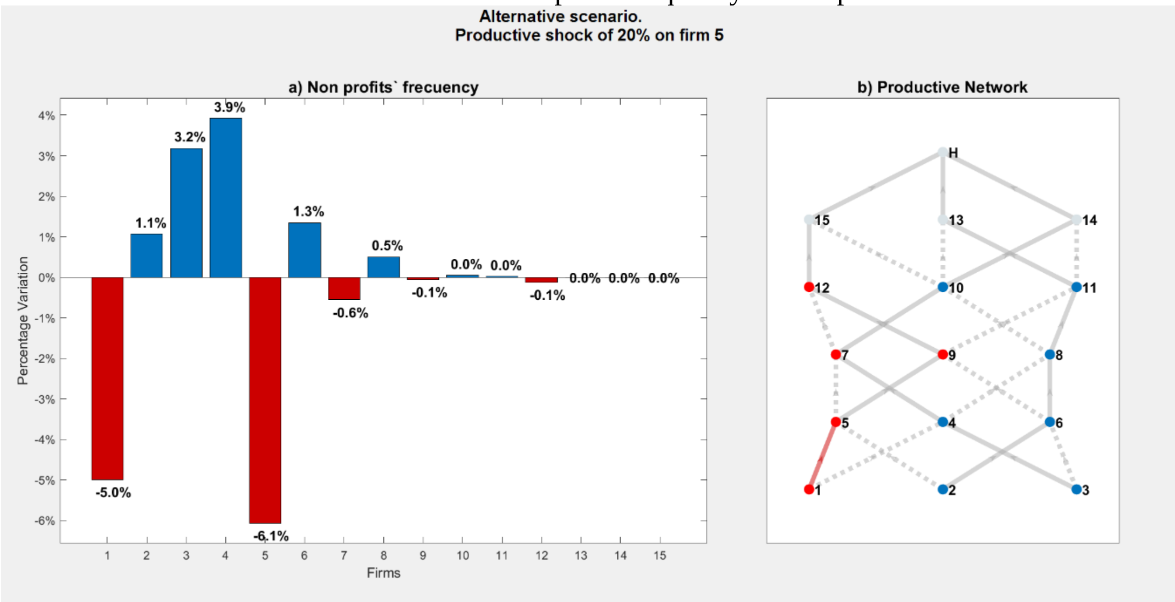
Figures. General Market Equilibrium on alternative scenarios
Variation of non-zero profit frequency. Initial period.

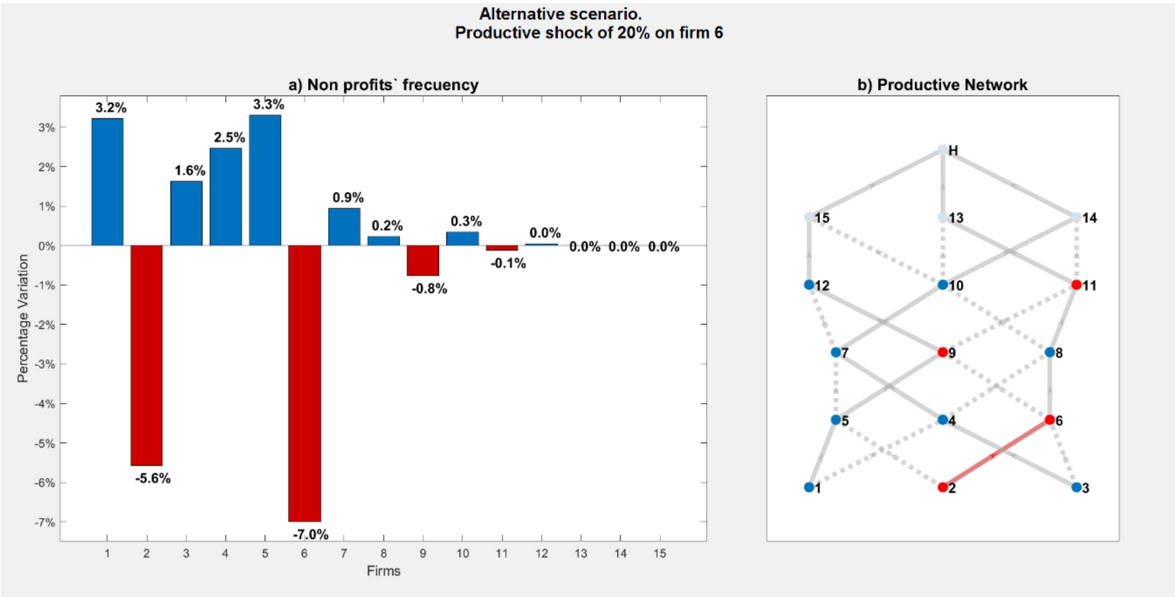




Note. Figure a) is a bar diagram that shows the percentage variation in the frequency with which firms earn non-negative profits. This frequency is computed based on the empirical distribution function of the GME before and after the productivity shock. The red-blue color gradation indicates the sign of the variation. The color red indicates a reduction in the frequency, while a blue coloration denotes an increase in the frequency. Figure b) replicates this color grading on the nodes in the production network and outlines a path with the nodes that exhibit the largest negative variation.

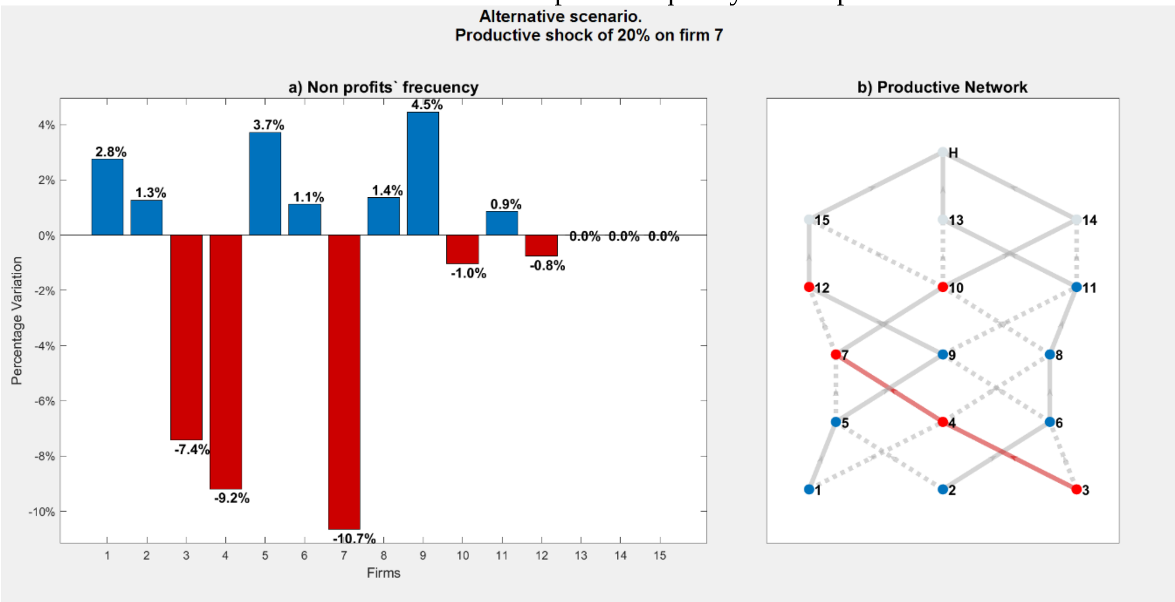
Figures. General Market Equilibrium on alternative scenarios
Variation of non-zero profit frequency. Initial period.

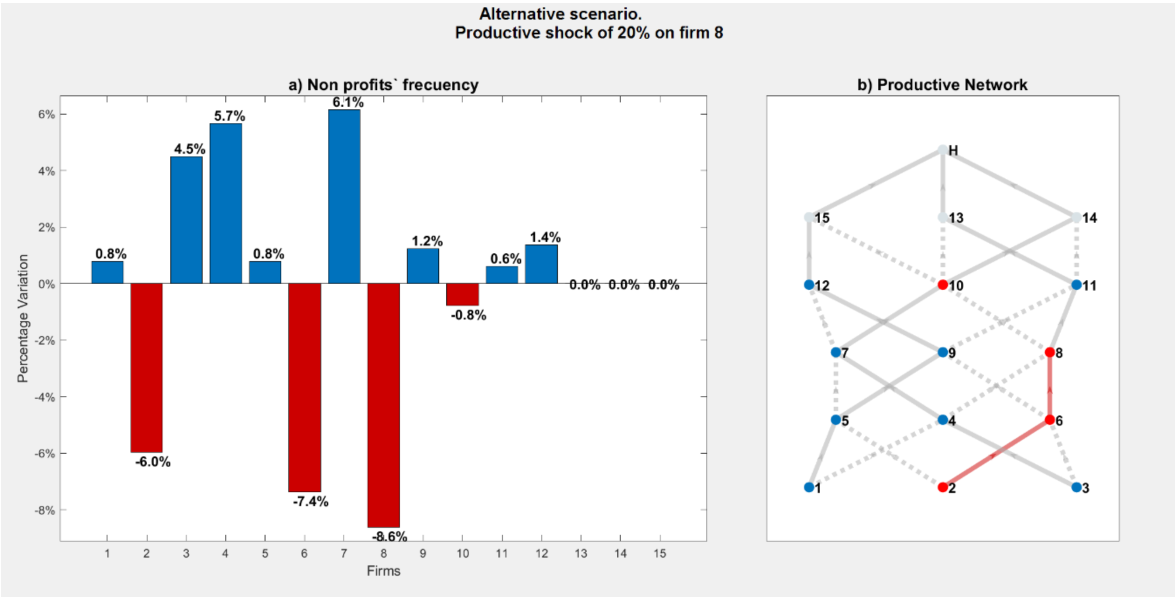




Note. Figure a) is a bar diagram that shows the percentage variation in the frequency with which firms earn non-negative profits. This frequency is computed based on the empirical distribution function of the GME before and after the productivity shock. The red-blue color gradation indicates the sign of the variation. The color red indicates a reduction in the frequency, while a blue coloration denotes an increase in the frequency. Figure b) replicates this color grading on the nodes in the production network and outlines a path with the nodes that exhibit the largest negative variation.

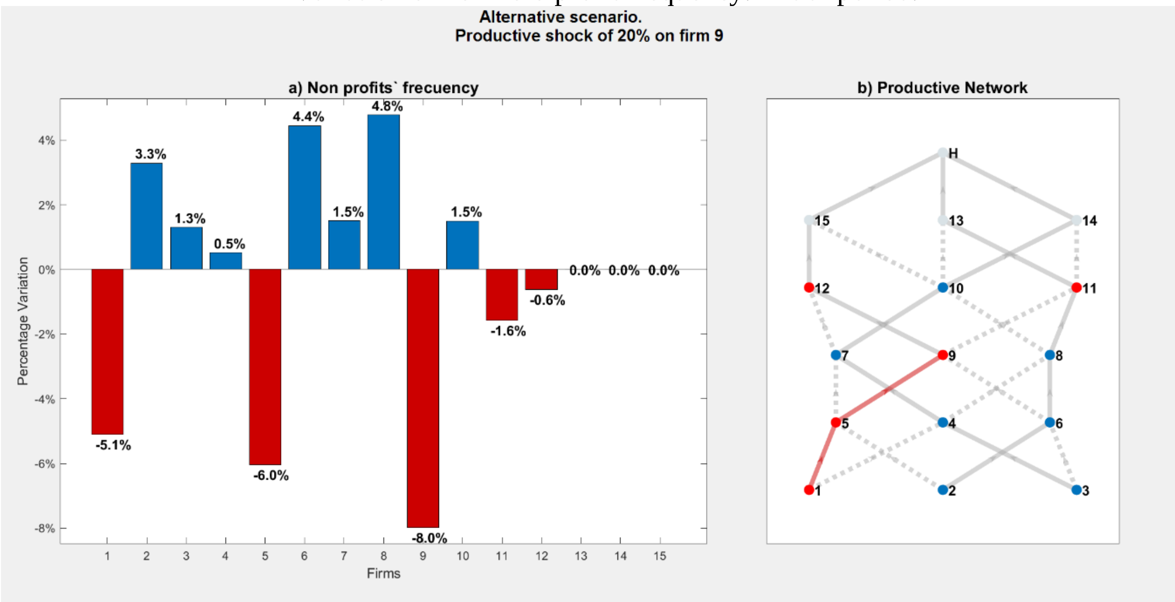
Figures. General Market Equilibrium on alternative scenarios
Variation of non-zero profit frequency. Initial period.

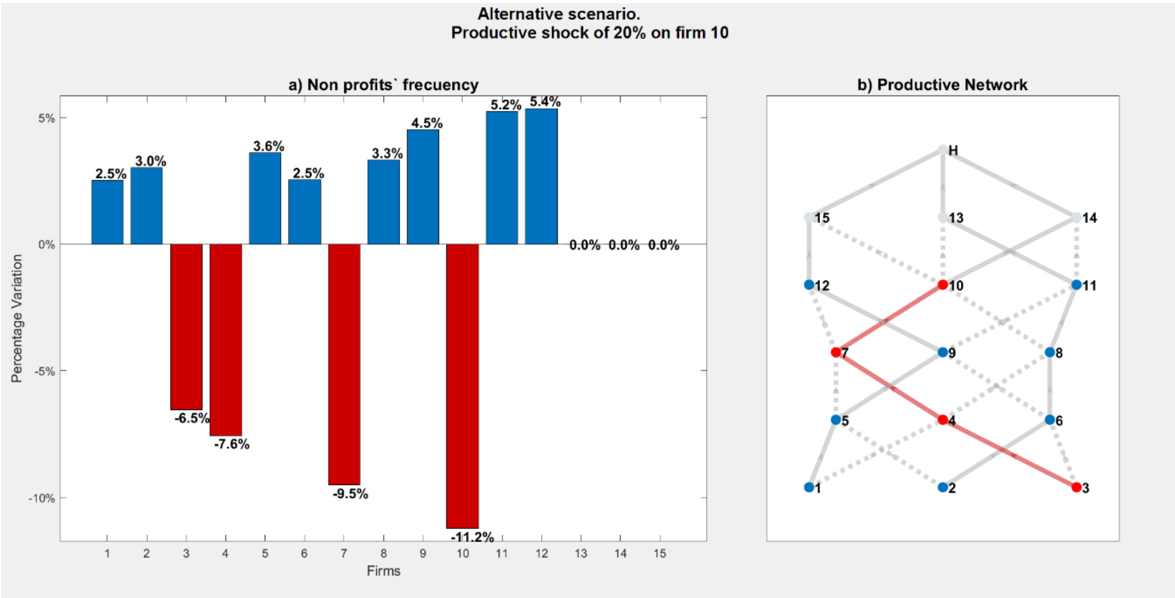




Note. Figure a) is a bar diagram that shows the percentage variation in the frequency with which firms earn non-negative profits. This frequency is computed based on the empirical distribution function of the GME before and after the productivity shock. The red-blue color gradation indicates the sign of the variation. The color red indicates a reduction in the frequency, while a blue coloration denotes an increase in the frequency. Figure b) replicates this color grading on the nodes in the production network and outlines a path with the nodes that exhibit the largest negative variation.

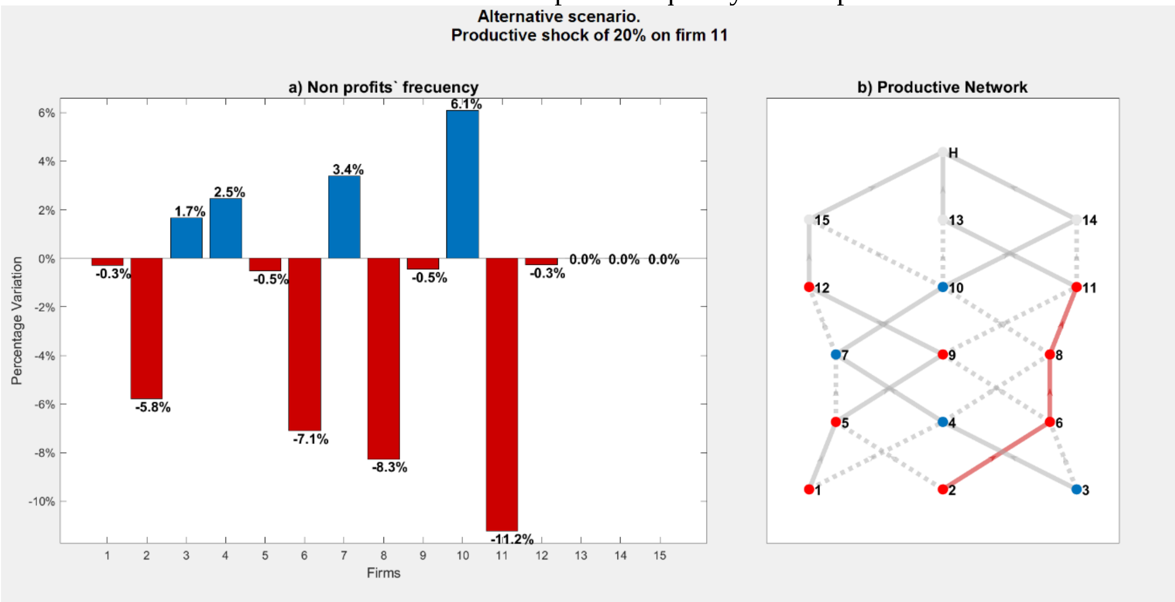
Figures. General Market Equilibrium on alternative scenarios
Variation of non-zero profit frequency. Initial period.

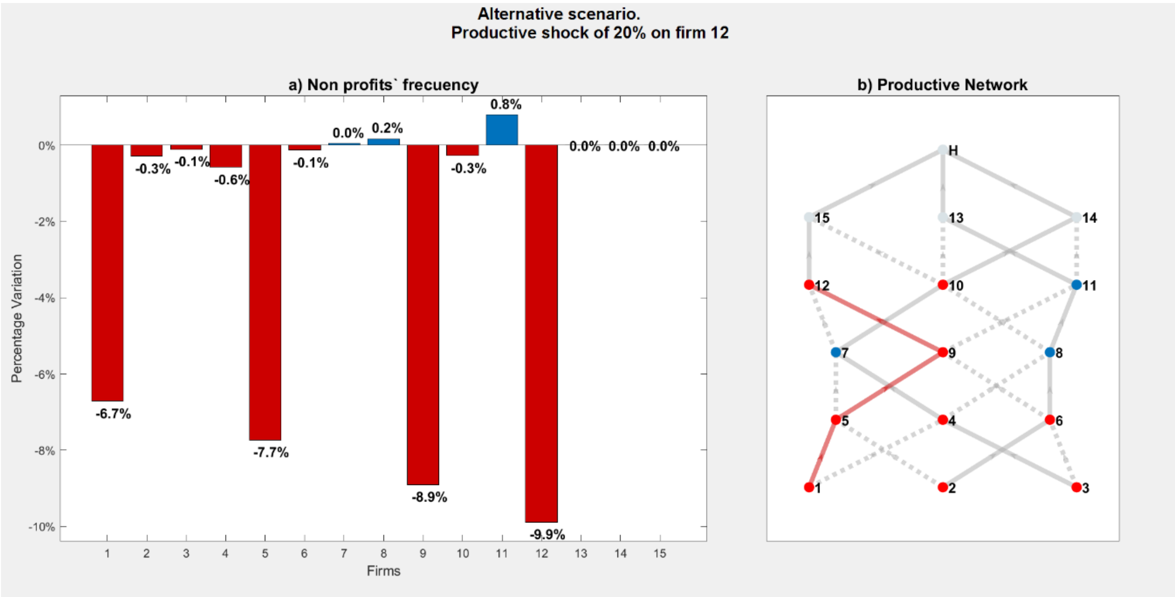




Note. Figure a) is a bar diagram that shows the percentage variation in the frequency with which firms earn non-negative profits. This frequency is computed based on the empirical distribution function of the GME before and after the productivity shock. The red-blue color gradation indicates the sign of the variation. The color red indicates a reduction in the frequency, while a blue coloration denotes an increase in the frequency. Figure b) replicates this color grading on the nodes in the production network and outlines a path with the nodes that exhibit the largest negative variation.

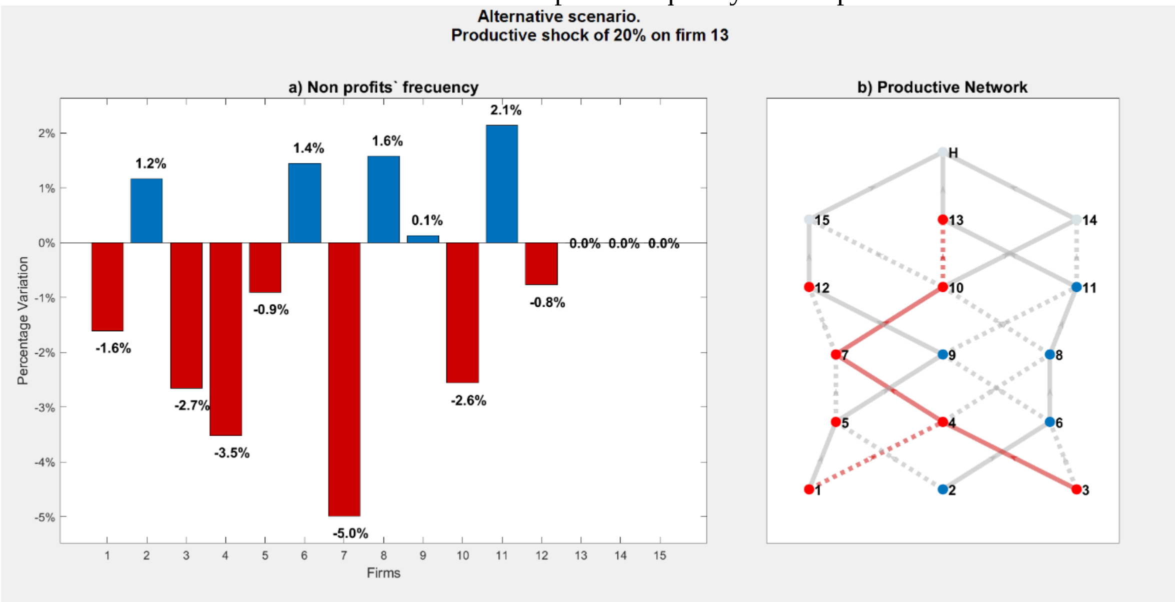
Figures. General Market Equilibrium on alternative scenarios
Variation of non-zero profit frequency. Initial period.

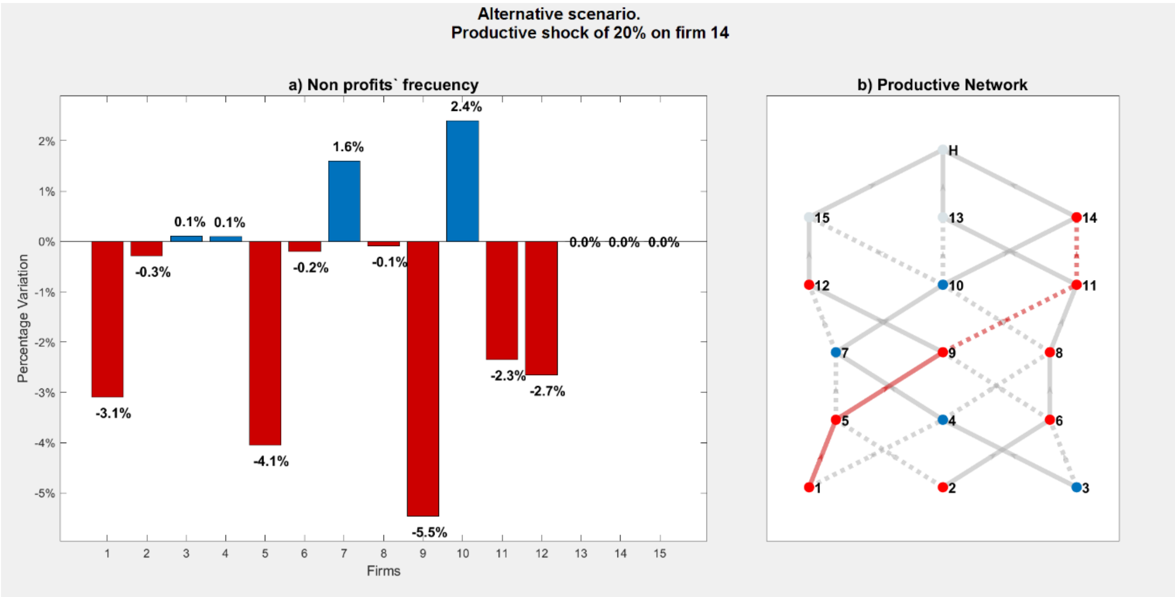




Note. Figure a) is a bar diagram that shows the percentage variation in the frequency with which firms earn non-negative profits. This frequency is computed based on the empirical distribution function of the GME before and after the productivity shock. The red-blue color gradation indicates the sign of the variation. The color red indicates a reduction in the frequency, while a blue coloration denotes an increase in the frequency. Figure b) replicates this color grading on the nodes in the production network and outlines a path with the nodes that exhibit the largest negative variation.

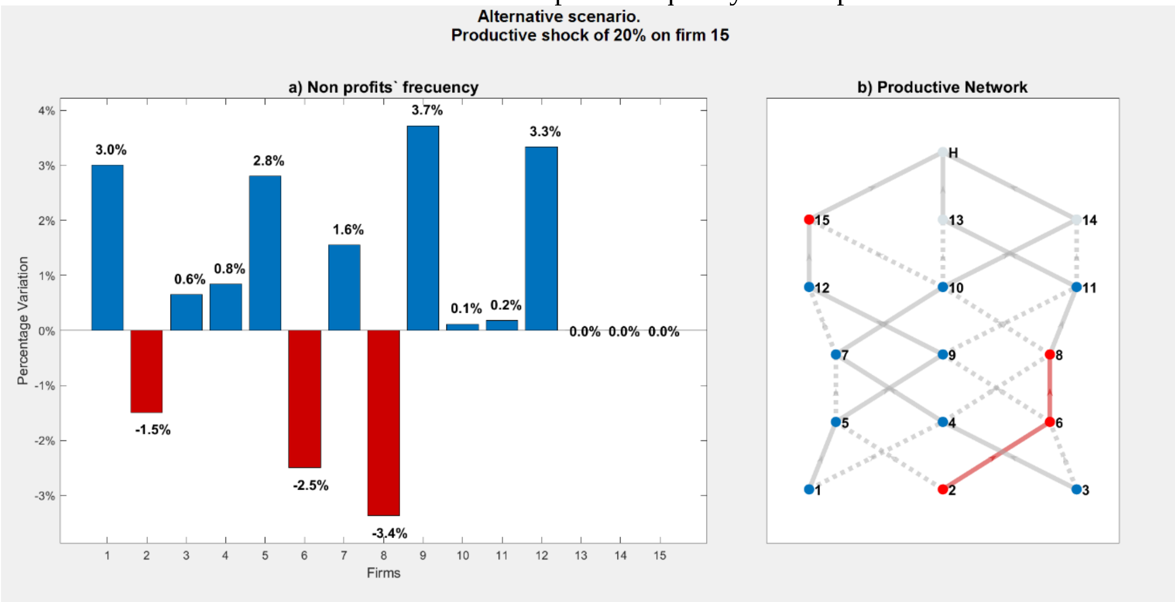
Figures. General Market Equilibrium on alternative scenarios
Variation of non-zero profit frequency. Initial period.





Note. Figure a) is a bar diagram that shows the percentage variation in the frequency with which firms earn non-negative profits. This frequency is computed based on the empirical distribution function of the GME before and after the productivity shock. The red-blue color gradation indicates the sign of the variation. The color red indicates a reduction in the frequency, while a blue coloration denotes an increase in the frequency. Figure b) replicates this color grading on the nodes in the production network and outlines a path with the nodes that exhibit the largest negative variation.

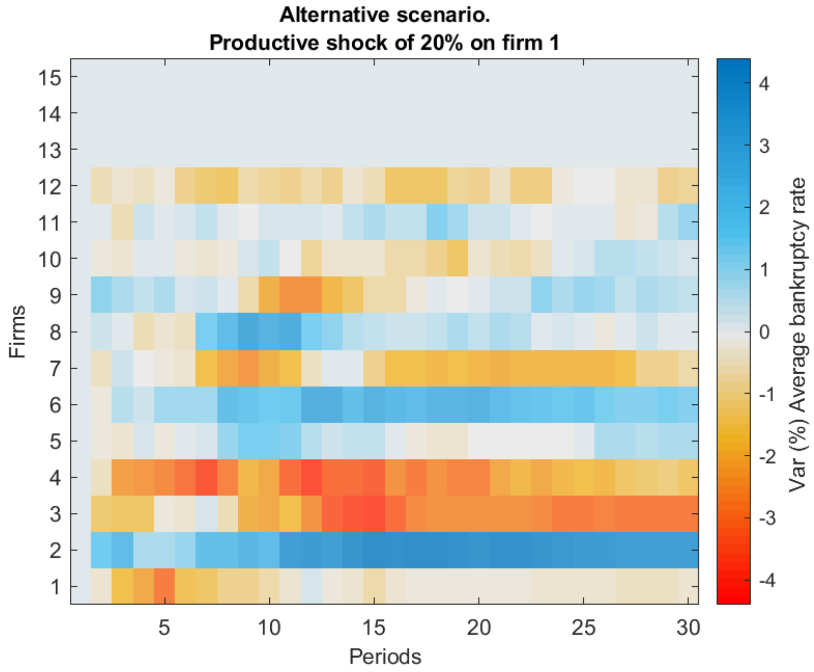
Figures. General Market Equilibrium on alternative scenarios
Variation of non-zero profit frequency. Initial period.

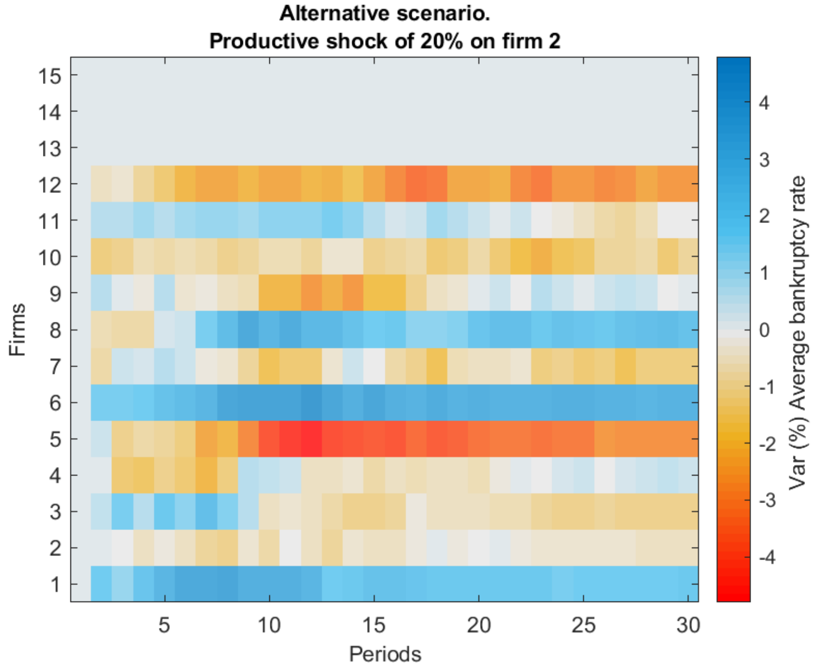


Note. Figure a) is a bar diagram that shows the percentage variation in the frequency with which firms earn non-negative profits. This frequency is computed based on the empirical distribution function of the GME before and after the productivity shock. The red-blue color gradation indicates the sign of the variation. The color red indicates a reduction in the frequency, while a blue coloration denotes an increase in the frequency. Figure b) replicates this color grading on the nodes in the production network and outlines a path with the nodes that exhibit the largest negative variation.

G.3. Bankruptcy of firms

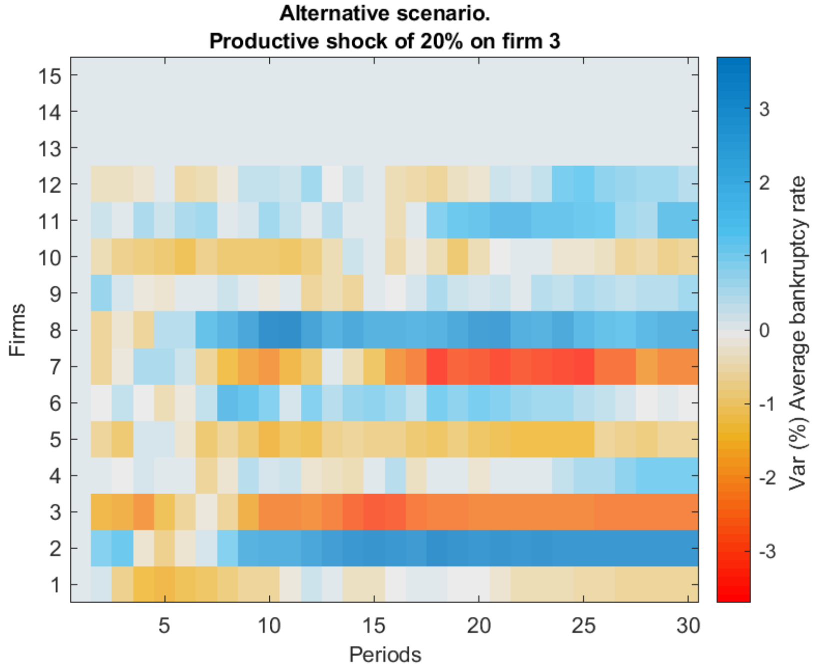
Figures. Bankruptcy of firms on alternative scenarios.
Variation of the average bankruptcy rate by firm. 30 time periods.

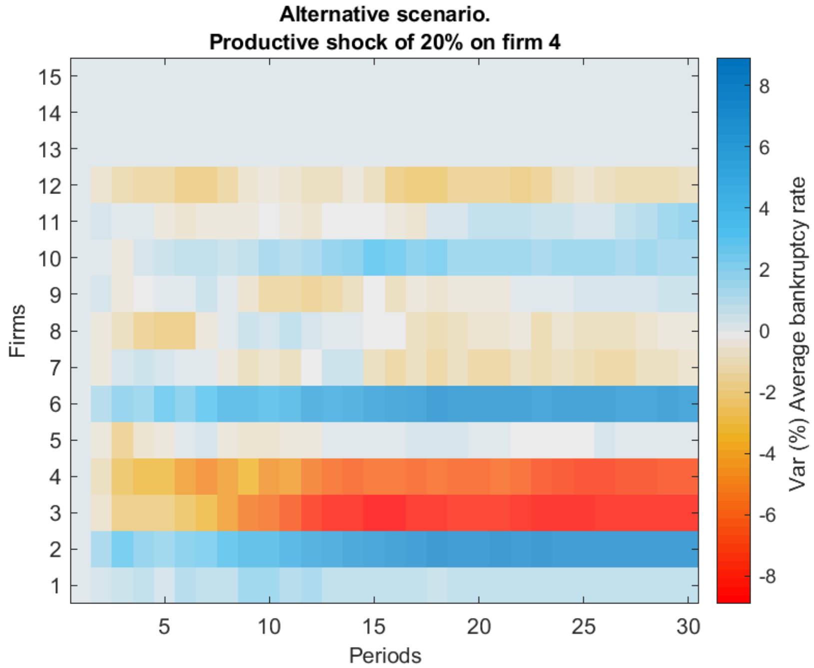




Note. This figure shows the percentage variation in the average bankruptcy rate of firms over a set of 1,000 random trials in a time horizon of 30 periods. Each row represents a firm and each column, a time period. The red-blue color gradation specifies the sign and magnitude of the variation. If colored red, then there is an increase in the rate of bankruptcy. The higher the intensity of the color red, the larger the rise in the bankruptcy rate. On the contrary, if colored blue, then there is a decrease in the rate of bankruptcy. The more intense the blue color is, the larger the decrease in the rate of bankruptcy.

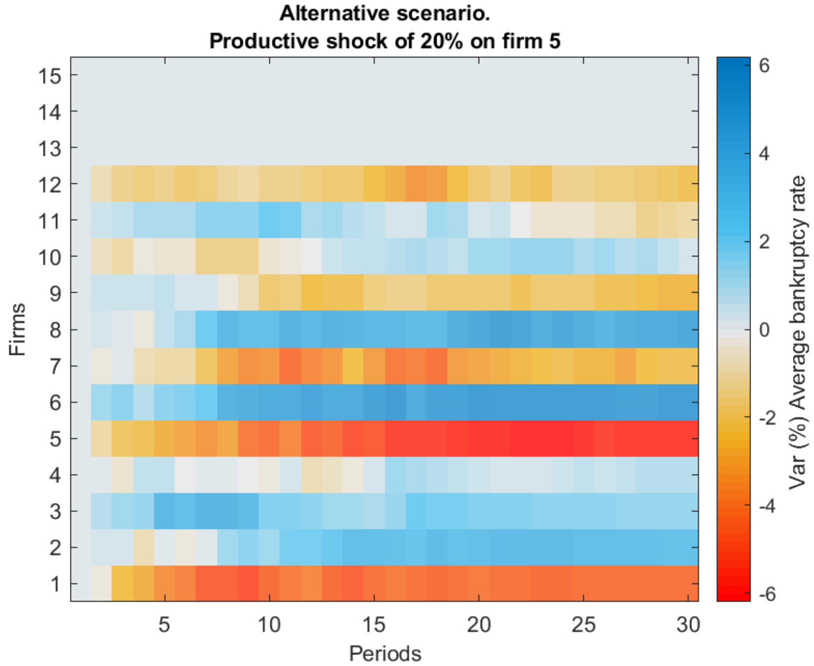
Figures. Bankruptcy of firms on alternative scenarios.
Variation of the average bankruptcy rate by firm. 30 time periods.

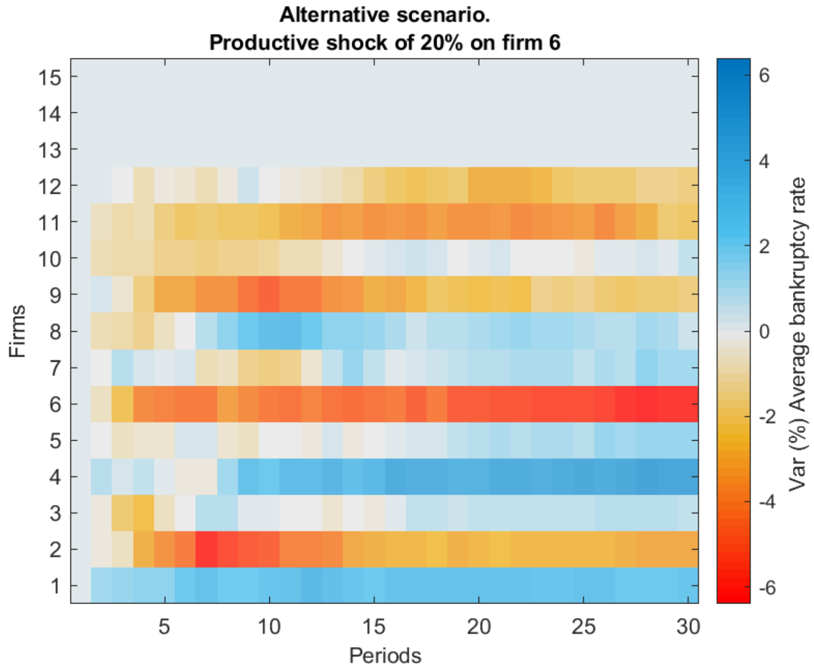




Note. This figure shows the percentage variation in the average bankruptcy rate of firms over a set of 1,000 random trials in a time horizon of 30 periods. Each row represents a firm and each column, a time period. The red-blue color gradation specifies the sign and magnitude of the variation. If colored red, then there is an increase in the rate of bankruptcy. The higher the intensity of the color red, the larger the rise in the bankruptcy rate. On the contrary, if colored blue, then there is a decrease in the rate of bankruptcy. The more intense the blue color is, the larger the decrease in the rate of bankruptcy.

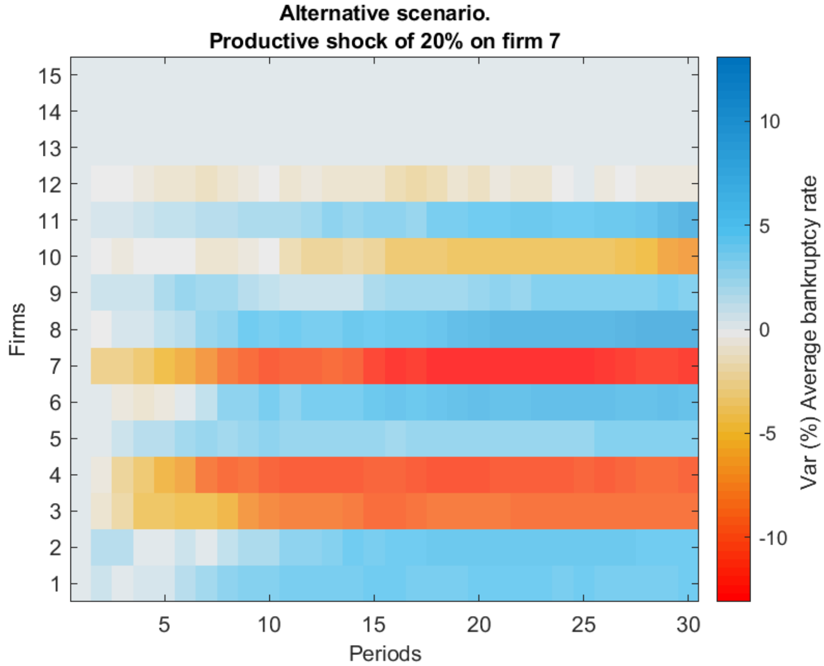
Figures. Bankruptcy of firms on alternative scenarios.
Variation of the average bankruptcy rate by firm. 30 time periods.

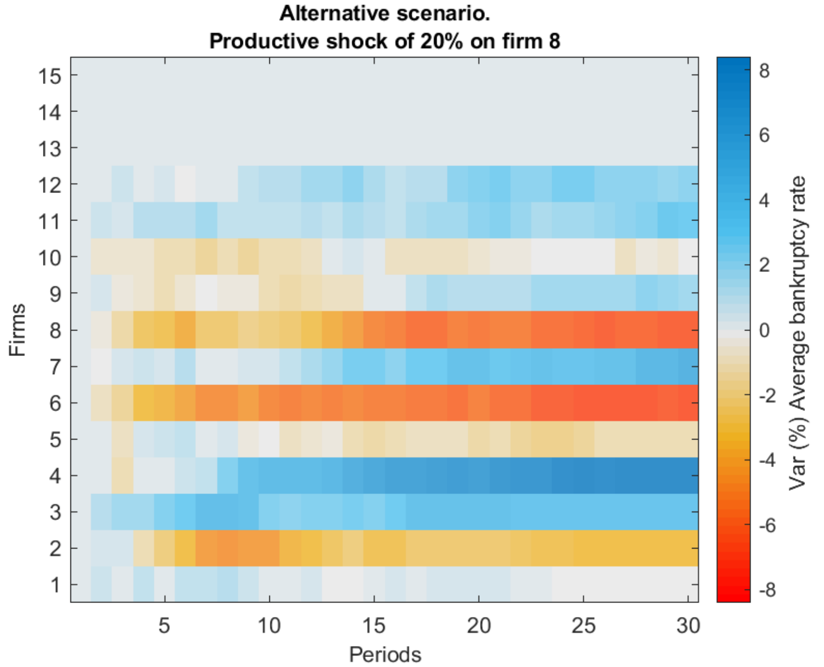




Note. This figure shows the percentage variation in the average bankruptcy rate of firms over a set of 1,000 random trials in a time horizon of 30 periods. Each row represents a firm and each column, a time period. The red-blue color gradation specifies the sign and magnitude of the variation. If colored red, then there is an increase in the rate of bankruptcy. The higher the intensity of the color red, the larger the rise in the bankruptcy rate. On the contrary, if colored blue, then there is a decrease in the rate of bankruptcy. The more intense the blue color is, the larger the decrease in the rate of bankruptcy.

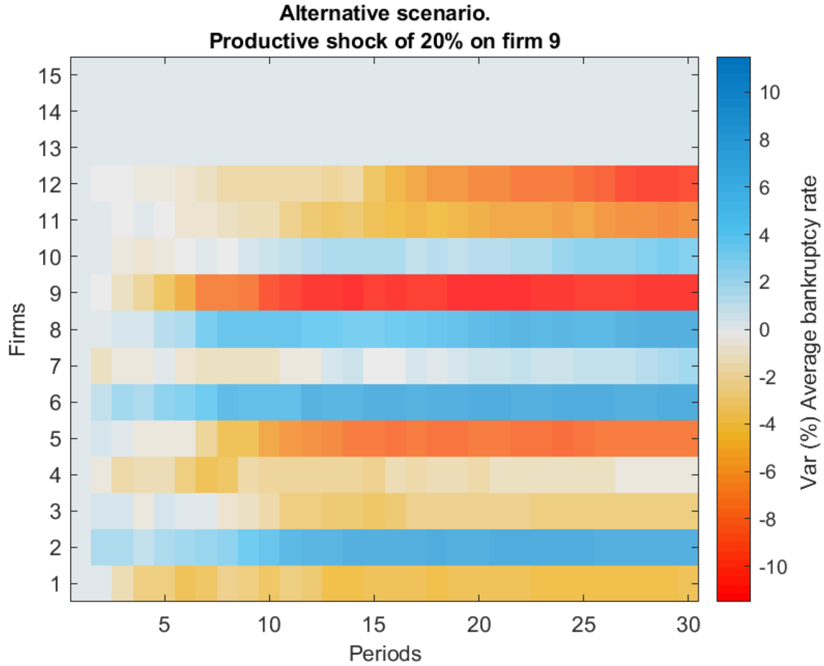
Figures. Bankruptcy of firms on alternative scenarios.
Variation of the average bankruptcy rate by firm. 30 time periods.

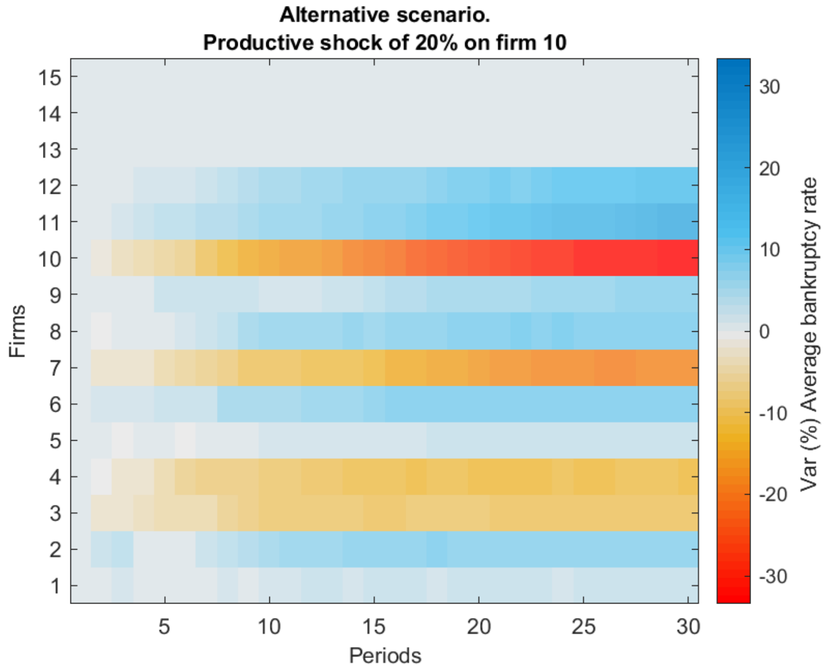




Note. This figure shows the percentage variation in the average bankruptcy rate of firms over a set of 1,000 random trials in a time horizon of 30 periods. Each row represents a firm and each column, a time period. The red-blue color gradation specifies the sign and magnitude of the variation. If colored red, then there is an increase in the rate of bankruptcy. The higher the intensity of the color red, the larger the rise in the bankruptcy rate. On the contrary, if colored blue, then there is a decrease in the rate of bankruptcy. The more intense the blue color is, the larger the decrease in the rate of bankruptcy.

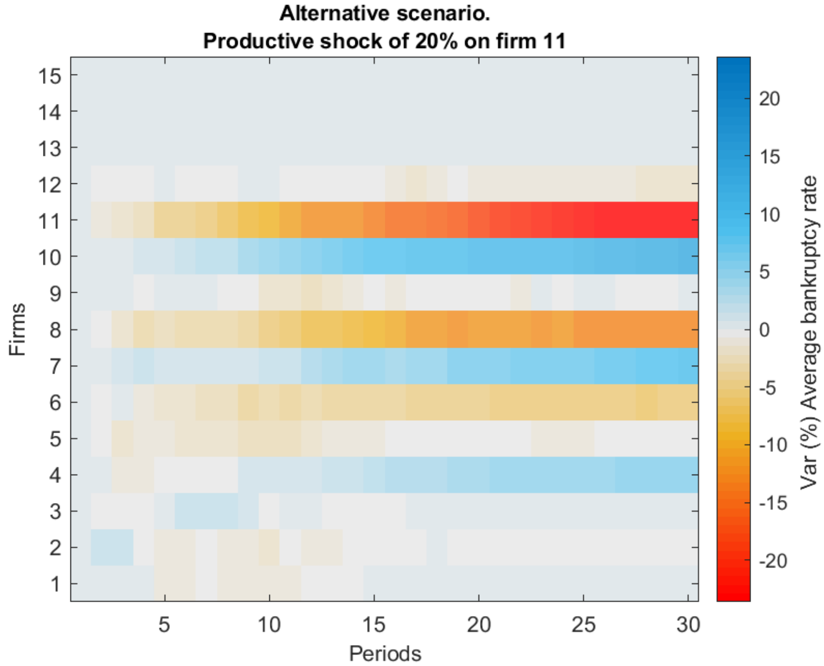
Figures. Bankruptcy of firms on alternative scenarios.
Variation of the average bankruptcy rate by firm. 30 time periods.

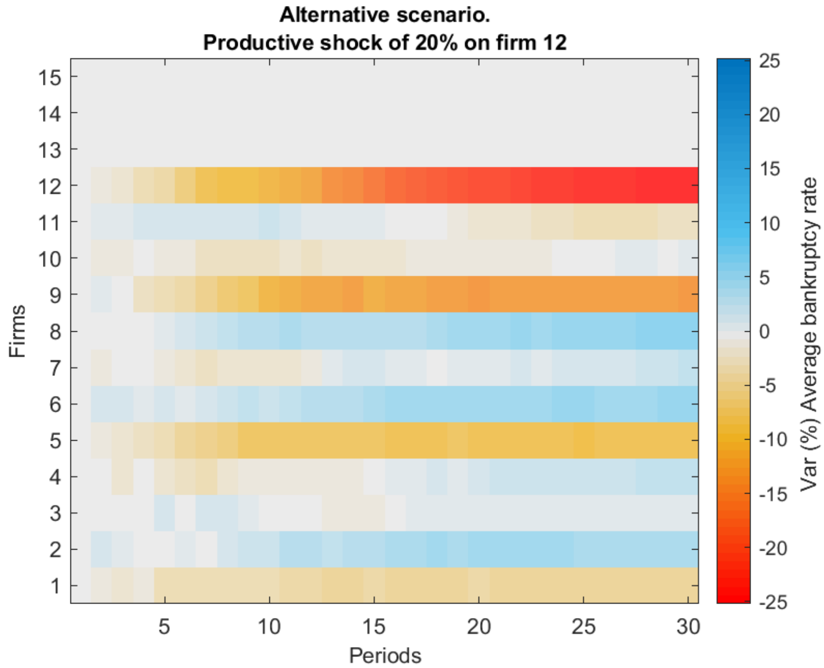




Note. This figure shows the percentage variation in the average bankruptcy rate of firms over a set of 1,000 random trials in a time horizon of 30 periods. Each row represents a firm and each column, a time period. The red-blue color gradation specifies the sign and magnitude of the variation. If colored red, then there is an increase in the rate of bankruptcy. The higher the intensity of the color red, the larger the rise in the bankruptcy rate. On the contrary, if colored blue, then there is a decrease in the rate of bankruptcy. The more intense the blue color is, the larger the decrease in the rate of bankruptcy.

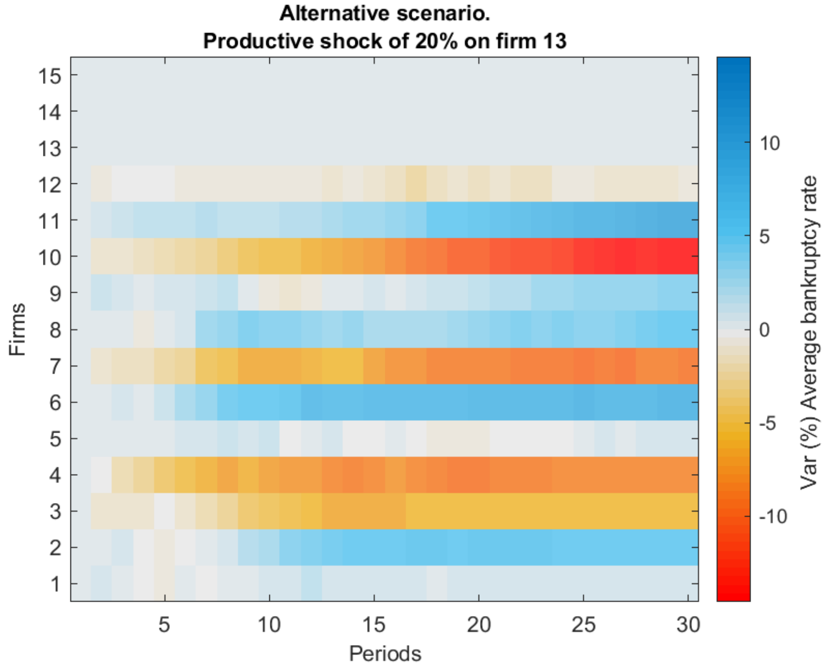
Figures. Bankruptcy of firms on alternative scenarios.
Variation of the average bankruptcy rate by firm. 30 time periods.

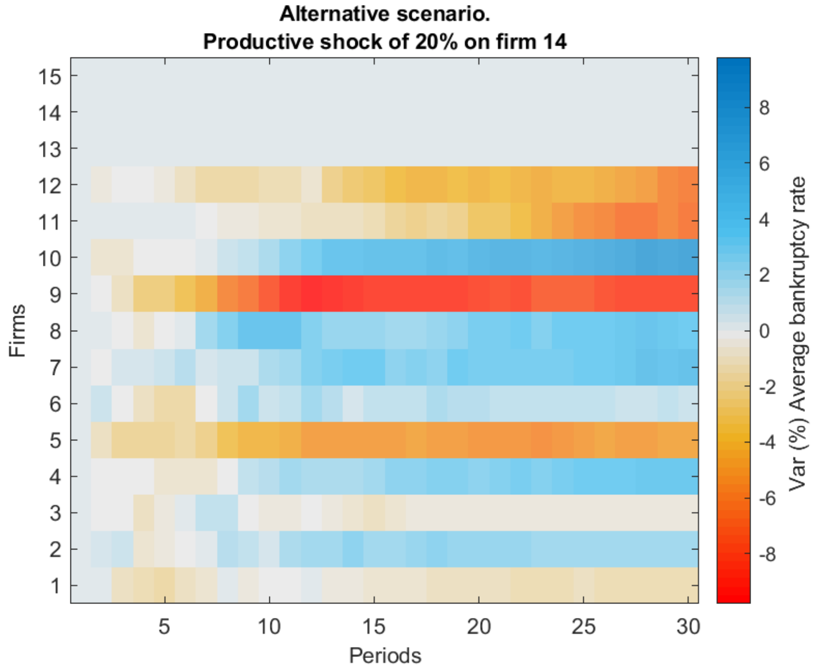




Note. This figure shows the percentage variation in the average bankruptcy rate of firms over a set of 1,000 random trials in a time horizon of 30 periods. Each row represents a firm and each column, a time period. The red-blue color gradation specifies the sign and magnitude of the variation. If colored red, then there is an increase in the rate of bankruptcy. The higher the intensity of the color red, the larger the rise in the bankruptcy rate. On the contrary, if colored blue, then there is a decrease in the rate of bankruptcy. The more intense the blue color is, the larger the decrease in the rate of bankruptcy.

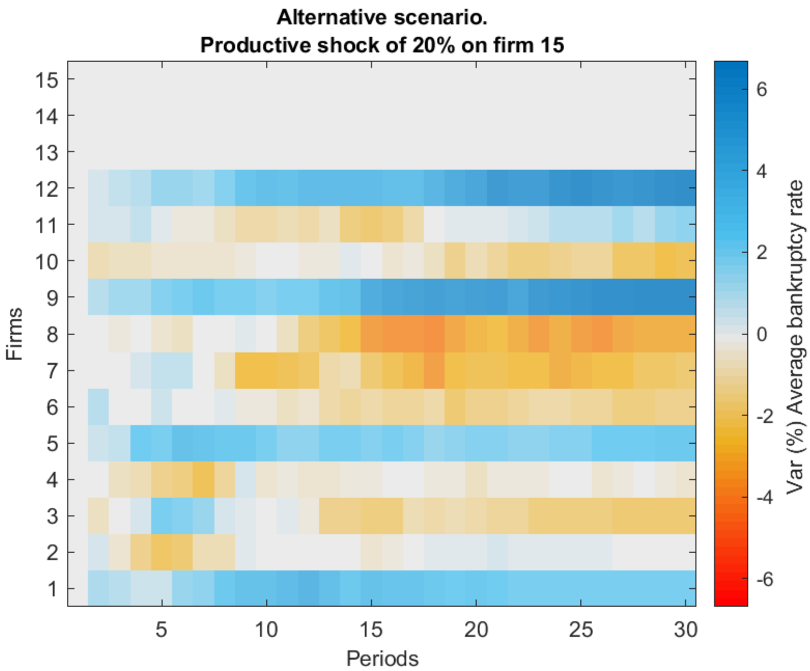
Figures. Bankruptcy of firms on alternative scenarios.
Variation of the average bankruptcy rate by firm. 30 time periods.





Note. This figure shows the percentage variation in the average bankruptcy rate of firms over a set of 1,000 random trials in a time horizon of 30 periods. Each row represents a firm and each column, a time period. The red-blue color gradation specifies the sign and magnitude of the variation. If colored red, then there is an increase in the rate of bankruptcy. The higher the intensity of the color red, the larger the rise in the bankruptcy rate. On the contrary, if colored blue, then there is a decrease in the rate of bankruptcy. The more intense the blue color is, the larger the decrease in the rate of bankruptcy.

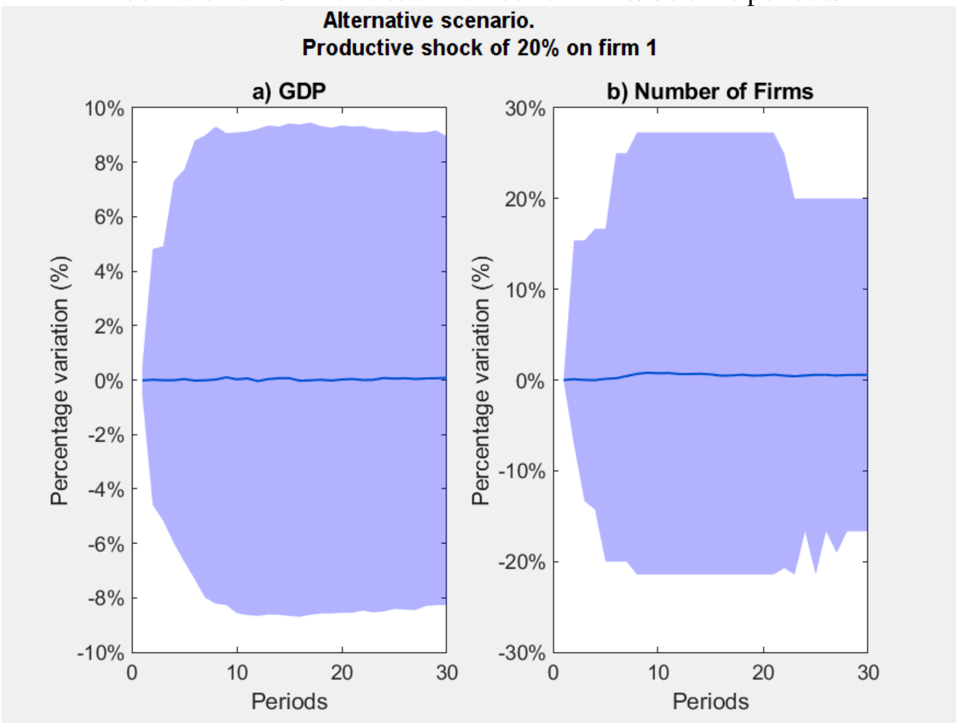
Figures. Bankruptcy of firms on alternative scenarios.
Variation of the average bankruptcy rate by firm. 30 time periods.

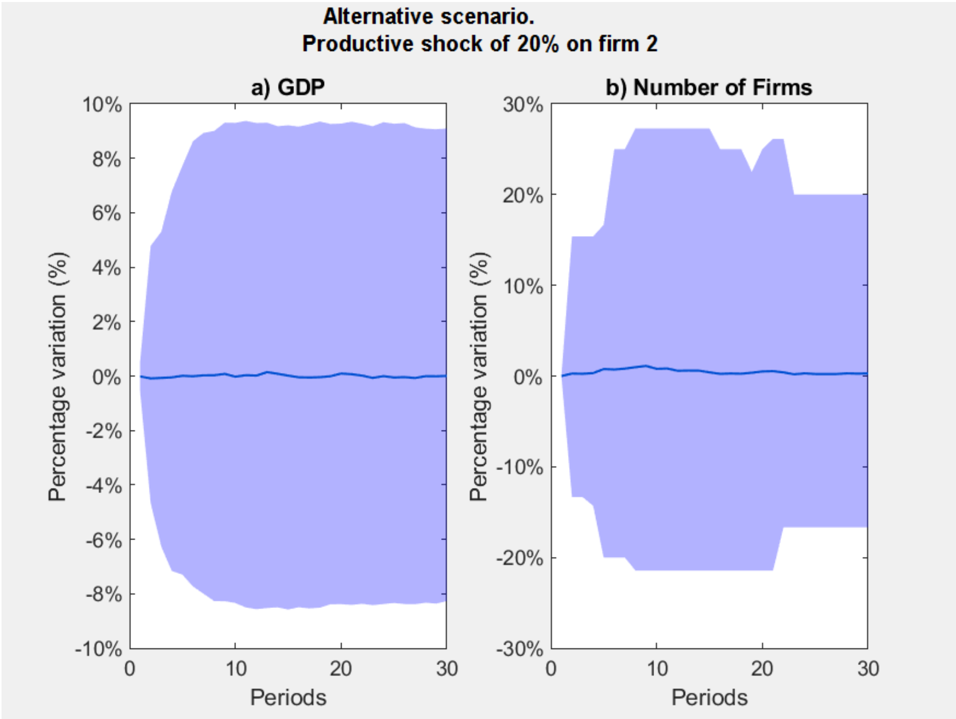


Note. This figure shows the percentage variation in the average bankruptcy rate of firms over a set of 1,000 random trials in a time horizon of 30 periods. Each row represents a firm and each column, a time period. The red-blue color gradation specifies the sign and magnitude of the variation. If colored red, then there is an increase in the rate of bankruptcy. The higher the intensity of the color red, the larger the rise in the bankruptcy rate. On the contrary, if colored blue, then there is a decrease in the rate of bankruptcy. The more intense the blue color is, the larger the decrease in the rate of bankruptcy.

G.4. GDP and number of firms

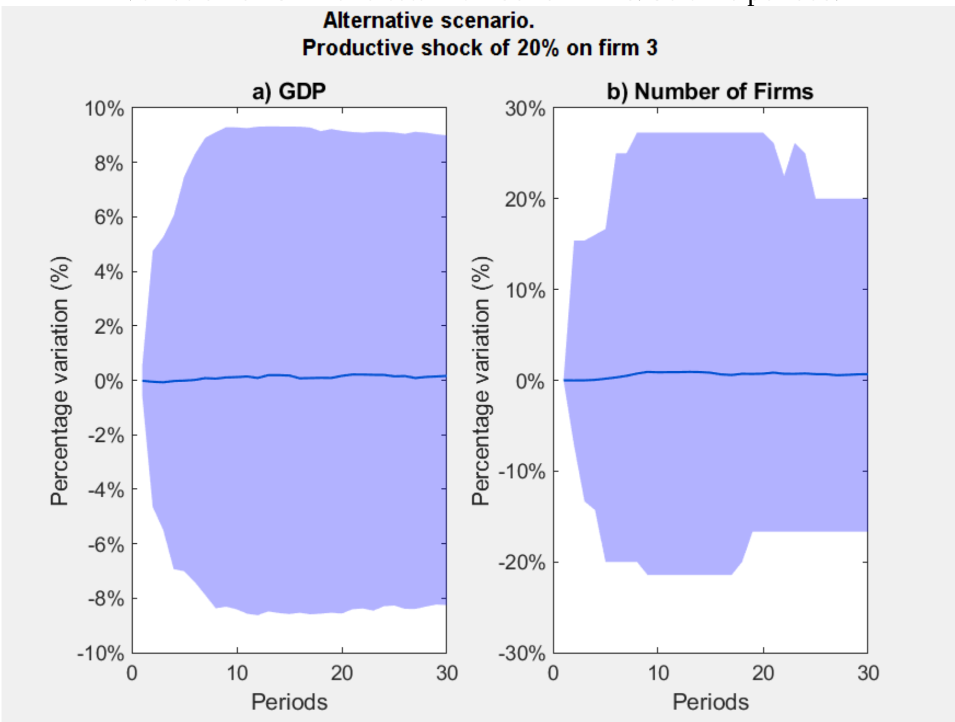
Figures. Bankruptcy of firms on alternative scenarios.
Variation of GDP and total number of firms. 30 time periods.

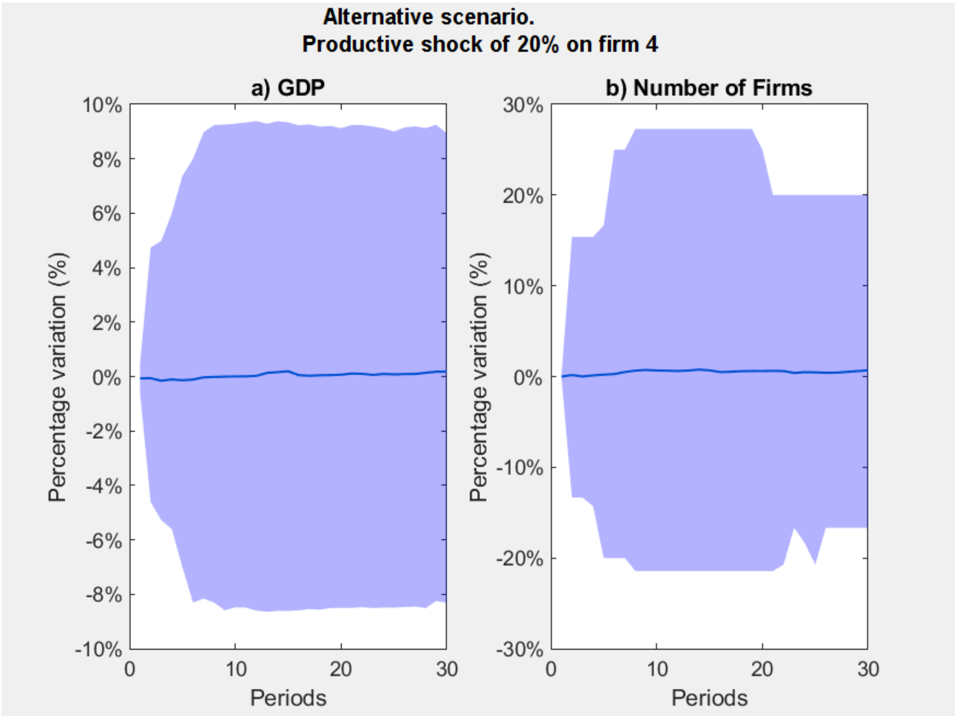




Note. Figures a) and b) show the percentage variation of the Gross Domestic Product and the total number of firms, respectively, over a set of 1,000 randomized trials in a time horizon of 30 periods. The thick line corresponds to the average percentage variation, while the blue bands represent 95% confidence intervals.

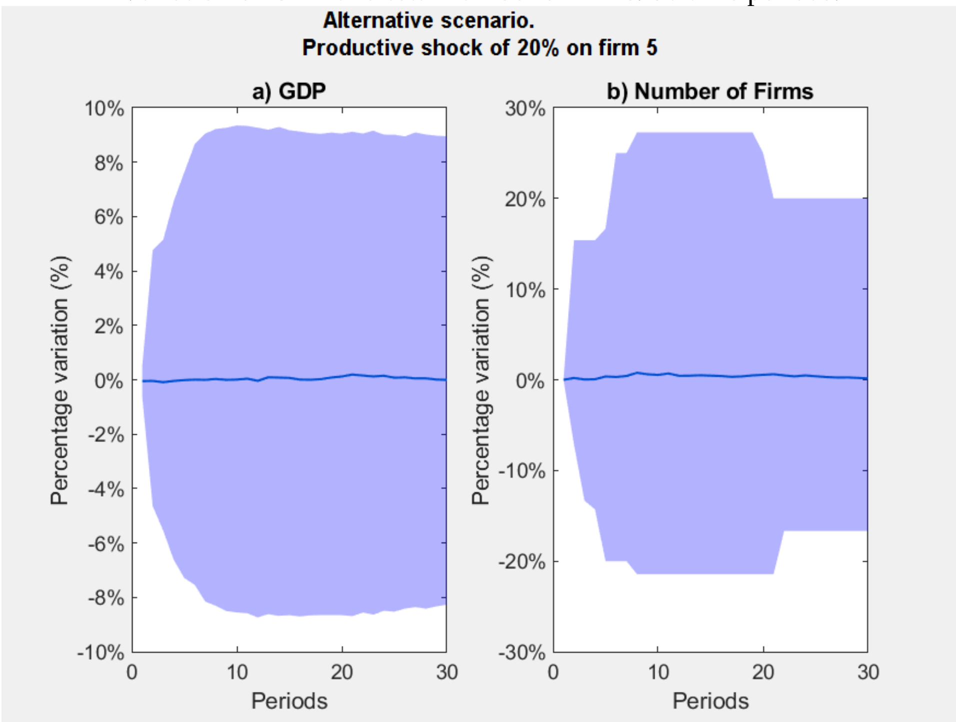
Figures. Bankruptcy of firms on alternative scenarios.
Variation of GDP and total number of firms. 30 time periods.

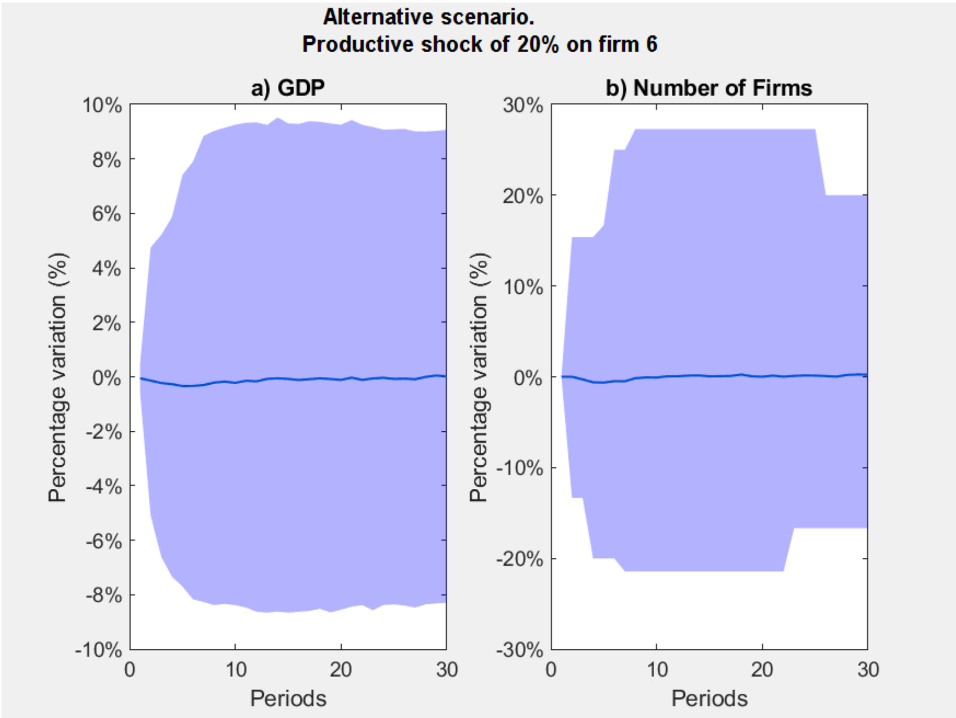




Note. Figures a) and b) show the percentage variation of the Gross Domestic Product and the total number of firms, respectively, over a set of 1,000 randomized trials in a time horizon of 30 periods. The thick line corresponds to the average percentage variation, while the blue bands represent 95% confidence intervals.

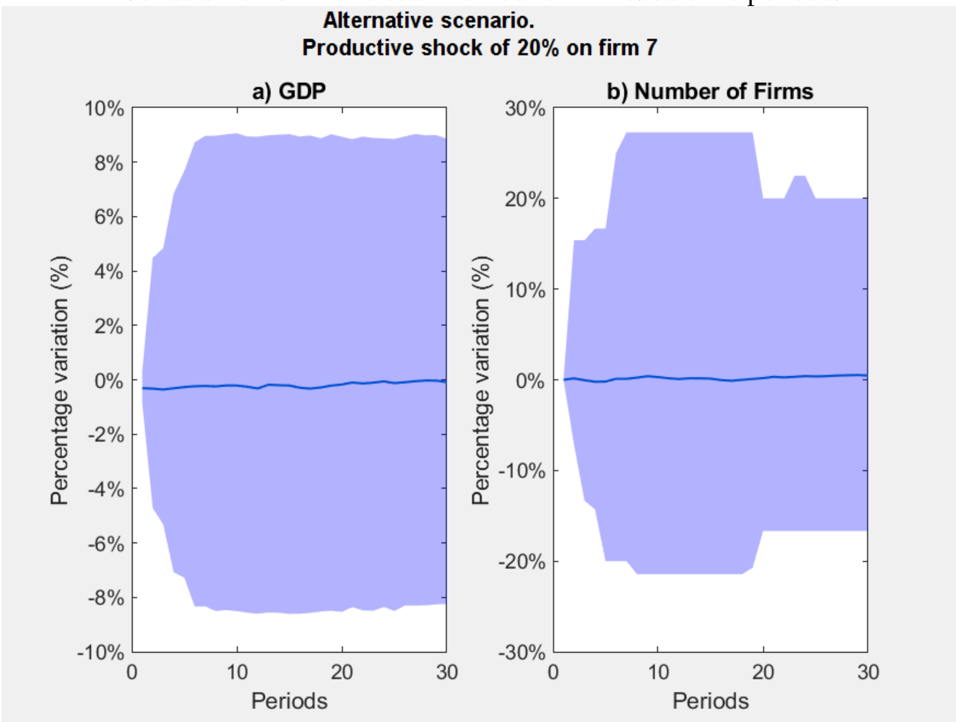
Figures. Bankruptcy of firms on alternative scenarios.
Variation of GDP and total number of firms. 30 time periods.

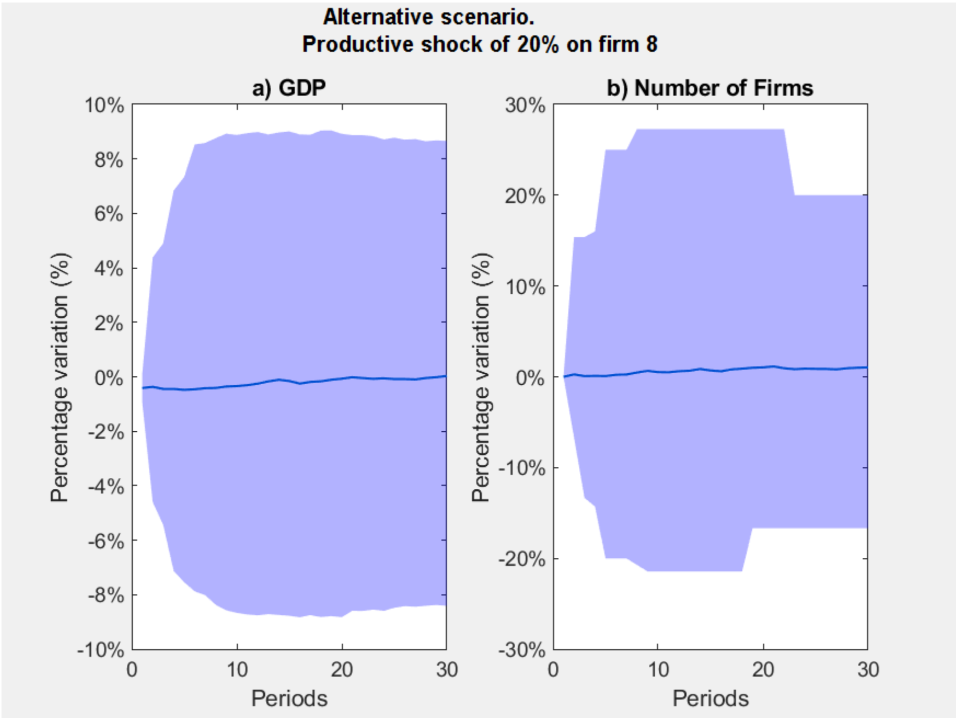




Note. Figures a) and b) show the percentage variation of the Gross Domestic Product and the total number of firms, respectively, over a set of 1,000 randomized trials in a time horizon of 30 periods. The thick line corresponds to the average percentage variation, while the blue bands represent 95% confidence intervals.

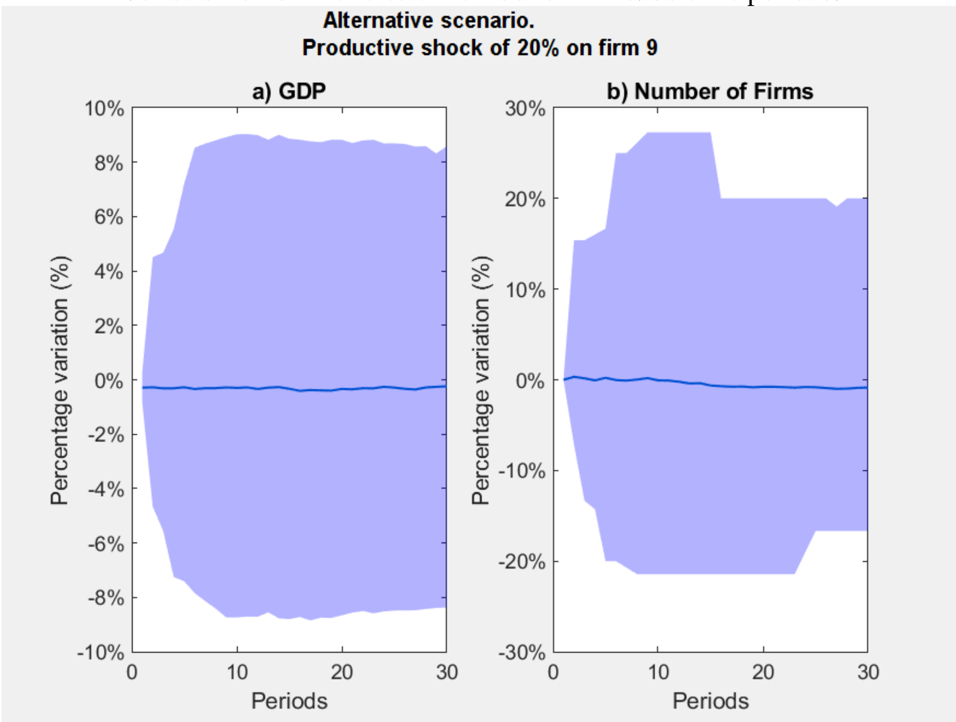
Figures. Bankruptcy of firms on alternative scenarios.
Variation of GDP and total number of firms. 30 time periods.

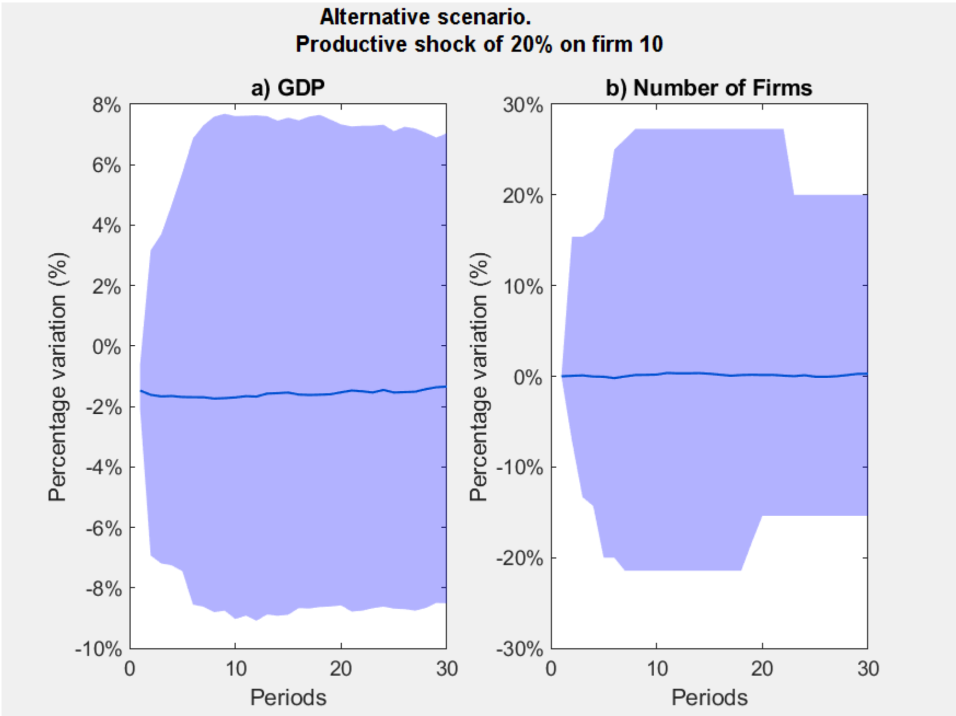




Note. Figures a) and b) show the percentage variation of the Gross Domestic Product and the total number of firms, respectively, over a set of 1,000 randomized trials in a time horizon of 30 periods. The thick line corresponds to the average percentage variation, while the blue bands represent 95% confidence intervals.

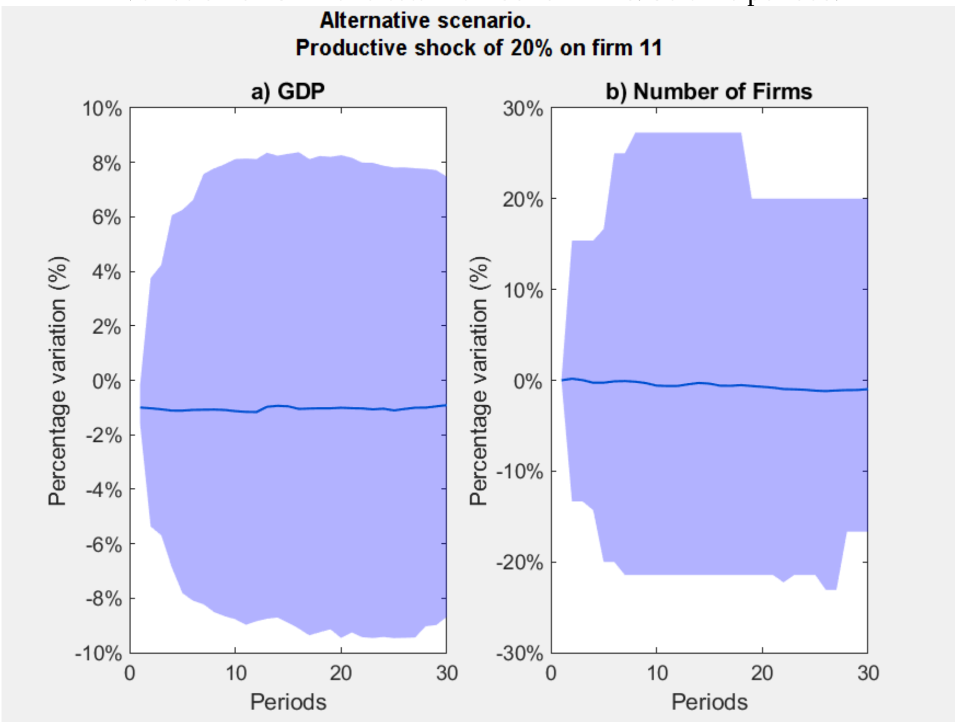
Figures. Bankruptcy of firms on alternative scenarios.
Variation of GDP and total number of firms. 30 time periods.

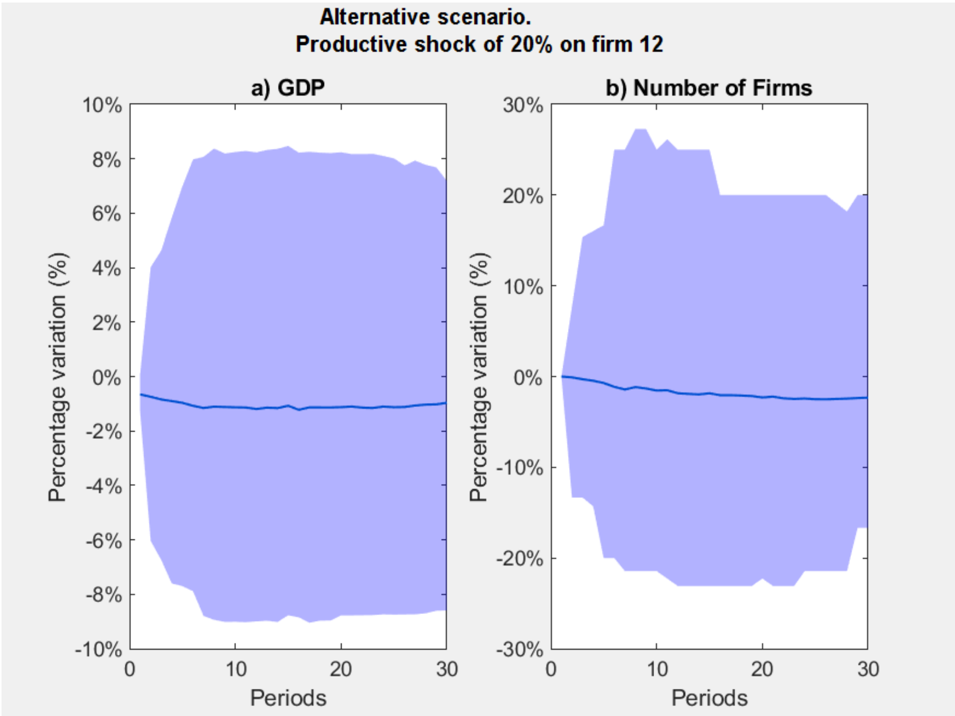




Note. Figures a) and b) show the percentage variation of the Gross Domestic Product and the total number of firms, respectively, over a set of 1,000 randomized trials in a time horizon of 30 periods. The thick line corresponds to the average percentage variation, while the blue bands represent 95% confidence intervals.

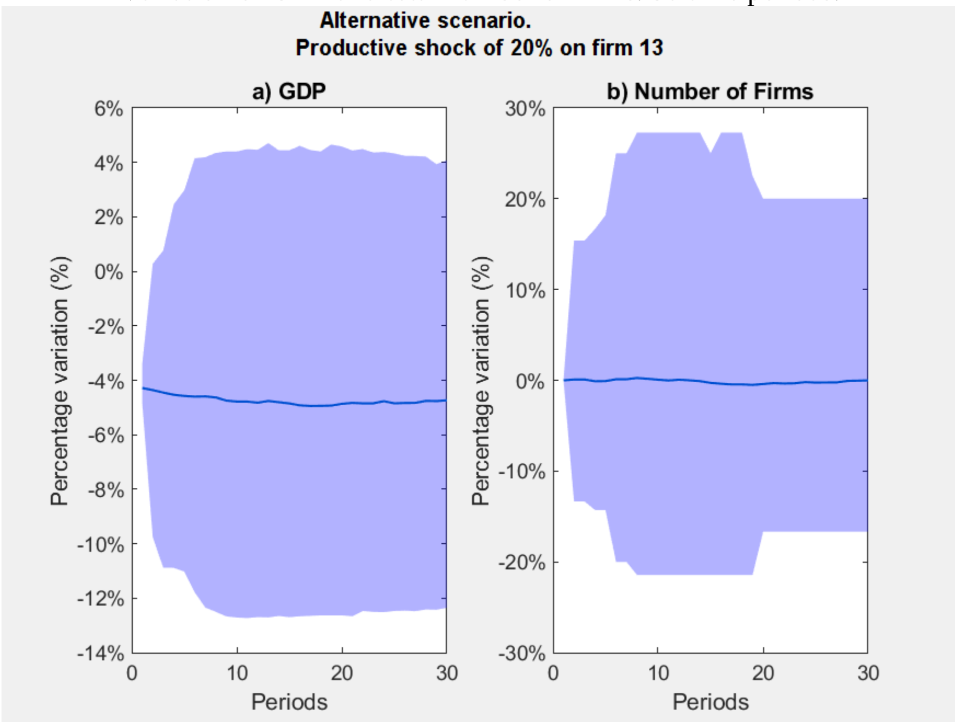
Figures. Bankruptcy of firms on alternative scenarios.
Variation of GDP and total number of firms. 30 time periods.

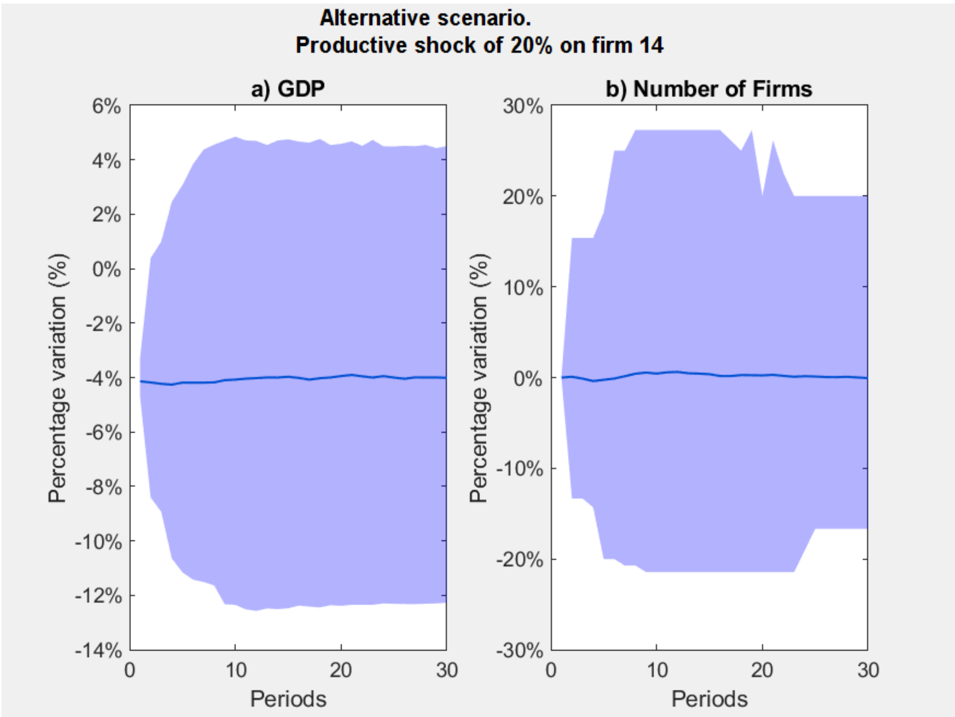




Note. Figures a) and b) show the percentage variation of the Gross Domestic Product and the total number of firms, respectively, over a set of 1,000 randomized trials in a time horizon of 30 periods. The thick line corresponds to the average percentage variation, while the blue bands represent 95% confidence intervals.

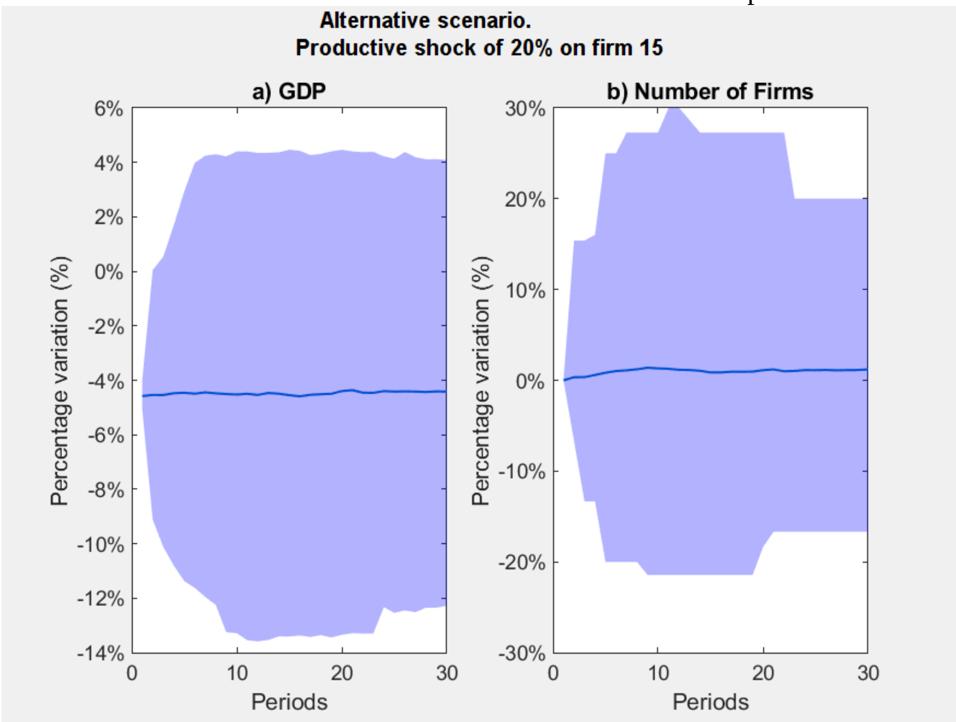
Figures. Bankruptcy of firms on alternative scenarios.
Variation of GDP and total number of firms. 30 time periods.





Note. Figures a) and b) show the percentage variation of the Gross Domestic Product and the total number of firms, respectively, over a set of 1,000 randomized trials in a time horizon of 30 periods. The thick line corresponds to the average percentage variation, while the blue bands represent 95% confidence intervals.

Figures. Bankruptcy of firms on alternative scenarios.
Variation of GDP and total number of firms. 30 time periods.



Note. Figures a) and b) show the percentage variation of the Gross Domestic Product and the total number of firms, respectively, over a set of 1,000 randomized trials in a time horizon of 30 periods. The thick line corresponds to the average percentage variation, while the blue bands represent 95% confidence intervals.

References

Acemoglu, Daron, Ufuk Akcigit, and William Kerr. 2015. “Networks and the Macroeconomy: An Empirical Exploration.” *NBER Macro Conference*, no. June: 1–27. doi:10.2139/ssrn.2632793.

Acemoglu, Daron, David Autor, David Dorn, Gordon H Hanson, Brendan Price, Daron Acemoglu, David Autor, Gordon H Hanson, and Brendan Price. 2016. “Import Competition and the Great US Employment Sag of the 2000s.” *Journal of Labor Economics* 34 (January).

Acemoglu, Daron, M. Carvalho, Vasco, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. 2012. “The Network Origins of Aggregate Fluctuations.” *Econometrica* 80 (5): 1977–2016. doi:10.3982/ECTA9623.

Acemoglu, Daron, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. 2010. “Cascades in Networks and Aggregate Volatility.” http://www.nber.org/papers/w16516.

Acemoglu, Daron, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. 2016. “Networks, Shocks, and Systemic Risk.” *The Oxford Handbook of the Economics of Networks*, no. January: 569–607. doi:10.3386/w20931.

———. 2017. “Microeconomic Origins of Macroeconomic Tail Risks.” *American Economic Review* 107 (1): 54–108. doi:10.1257/aer.20151086.

Allingham, Mg, and A Sandmo. 1972. “Income Tax Evasion: A Theoretical Analysis.” *Journal of Public Economics* 1: 323–38.

Alm, James, Todd Cherry, Michael Jones, and Michael McKee. 2010. “Taxpayer Information Assistance Services and Tax Compliance Behavior.” *Journal of Economic Psychology* 31 (4). Elsevier B.V.: 577–86. doi:10.1016/j.joep.2010.03.018.

Alm, James, Betty R. Jackson, and Michael McKee. 1992. “Institutional Uncertainty and Taxpayer Compliance.” *American Economic Review* 82 (4): 1018–26.

Andreoni, James, Brian Erard, and Jonathan Feinstein. 1998. “Tax Compliance.” *American Economic Association* 36 (2): 818–60.

Aobdia, Daniel, Judson Caskey, and N Bugra Ozel. 2014. “Inter-Industry Network Structure and the Cross-Predictability of Earnings and Stock Returns.” *Review of Accounting Studies* 19 (3): 1191–1224. doi:10.1007/s11142-014-9286-7.

Appelgren, Leif. 2008. “The Effect of Audit Strategy Information on Tax Compliance - an Empirical Study.” *EJournal of Tax Research* 6 (1): 67–81.

Ariel, Barak. 2012. “Deterrence and Moral Persuasion Effects on Corporate Tax Compliance:

Findings from a Randomized Controlled Trial.” *Criminology* 50 (1): 27–69. doi:10.1111/j.1745-9125.2011.00256.x.

Armstrong, Christopher S., Jennifer L. Blouin, and David F. Larcker. 2012. “The Incentives for Tax Planning.” *Journal of Accounting and Economics* 53 (1–2). Elsevier: 391–411. doi:10.1016/j.jacceco.2011.04.001.

Atalay, Enghin. 2017. “How Important Are Sectoral Shocks?” *American Economic Journal: Macroeconomics* 9 (4): 254–80. doi:10.1257/mac.20160353.

Baqaee, David. 2016. “Cascading Failures in Production Networks.” *Ssrn*. doi:10.2139/ssrn.2909868.

Baqaee, David Rezza. 2018. “Cascading Failures in Production Networks.” *Econometrica* 86 (5): 1819–38. doi:10.3982/ECTA15280.

Baqaee, David Rezza, and Emmanuel Farhi. 2019. “The Macroeconomic Impact of Microeconomic Shocks: Beyond Hulten’s Theorem.” *Econometrica* 87 (4): 1155–1203. doi:10.3982/ECTA15202.

Barile, Lory. 2012. “Does Tax Evasion Affect Frms Internal Control(q) Some Evidence from an Experimental Approach.” *Labsi - Experimental Economics Laboratory, University of Siena - Working Paper Series*, no. 39.

Barrot, Jean-Noël, and Julien Sauvagnat. 2016. “Input Specificity and the Propagation of Idiosyncratic Shocks in Production Networks.” *The Quarterly Journal of Economics* 131 (3): 1543–92. doi:10.1093/qje/qjw018.

Bartelme, Dominick, and Yuriy Gorodnichenko. 2015. “Linkages and Economic Development.” Cambridge, MA. doi:10.3386/w21251.

Bazart, C., and A. Bonein. 2014. “Reciprocal Relationships in Tax Compliance Decisions.” *Journal of Economic Psychology* 40. Elsevier B.V.: 83–102. doi:10.1016/j.joep.2012.10.002.

Becker, Gary S. 1968. “Crime and Punishment: An Economic Approach.” *Journal of Political Economy* 76 (2): 169–217. http://www.nber.org/chapters/c3625.pdf.

Bigio, Saki, and Jennifer La’O. 2016. “Financial Frictions in Production Networks.” *Working Paper*.

Bimpikis, Kostas, S Ehsani, and R Ilkiliç. 2014. “Cournot Competition in Networked Markets.” doi:10.1145/2600057.2600058.

Biswas, Rongili, Carla Marchese, and Fabio Privileggi. 2013. “Firm’s Tax Evasion in a Principal-Agent Model with Self-Protection.” *Journal of Economics/ Zeitschrift Fur Nationalokonomie* 110 (2): 125–40. doi:10.1007/s00712-012-0306-4.

Boehm, Christoph E., Aaron Flaaen, and Nitya Pandalai-Nayar. 2019. “Input Linkages and the Transmission of Shocks: Firm-Level Evidence from the 2011 Tōhoku Earthquake.” *The Review of Economics and Statistics* 101 (1): 60–75. doi:10.1162/rest_a_00750.

Bramoullé, Yann, Andrea Galeotti, and Brian Rogers, eds. 2016. *The Oxford Handbook of the Economics of Networks*. Vol. 1. Oxford University Press. doi:10.1093/oxfordhb/9780199948277.001.0001.

Carrillo, Paul, Dina Pomeranz, and Monica Singhal. 2017. “Dodging the Taxman: Firm Misreporting and Limits to Tax Enforcement.” *American Economic Journal: Applied Economics* 9 (2): 144–64. doi:10.1257/app.20140495.

Carvalho, Vasco M. 2014. “From Micro to Macro via Production Networks.” *Journal of Economic Perspectives* 28 (4): 23–48. doi:10.1257/jep.28.4.23.

Carvalho, Vasco M., and Xavier Gabaix. 2013. “The Great Diversification and Its Undoing.” *American Economic Review* 103 (5): 1697–1727. doi:10.1257/aer.103.5.1697.

Carvalho, Vasco M. 2010. “Aggregate Fluctuations and the Network Structure of Intersectoral Trade.” *Economics Working Papers 1206*. http://www.econ.ucdavis.edu/seminars/papers/337/3371.pdf.

Carvalho, Vasco M, Makoto Nirei, Yukiko U Saito, and Alireza Tahbaz-Salehi. 2016. “Supply Chain Disruptions : Evidence from the Great East Japan Earthquake.”

Chen, K P, and C Y C Chu. 2005. “Internal Control versus External Manipulation: A Model of Corporate Income Tax Evasion.” *Rand Journal of Economics* 36 (1): 151–64.

Coleman, Steve. 1996. “The Minnesota Income Tax Compliance Experiment: State Tax Results.” *Minnesota Department of Revenue*. http://mpra.ub.uni-muenchen.de/4827/.

Coricelli, Giorgio, Elena Rusconi, and Marie Claire Villeval. 2014. “Tax Evasion and Emotions: An Empirical Test of Re-Integrative Shaming Theory.” *Journal of Economic Psychology* 40. Elsevier B.V.: 49–61. doi:10.1016/j.joep.2012.12.002.

Cowell, Frank A. 2004. “Carrots and Sticks in Enforcement.” *The Crisis in Tax Administration*, 230–75.

Crocker, Keith J., and Joel Slemrod. 2005. “Corporate Tax Evasion with Agency Costs.” *Journal of Public Economics* 89 (9–10): 1593–1610. doi:10.1016/j.jpubeco.2004.08.003.

Desai, M, A Dyck, and L Zingales. 2007. “Theft and Taxes☆.” *Journal of Financial Economics* 84 (3): 591–623. doi:10.1016/j.jfineco.2006.05.005.

Desai, Mihir A, and Dhammika Dharmapala. 2006. “Corporate Tax Avoidance and High-Powered Incentives.” *Journal of Financial Economics* 79 (1): 145–79. doi:10.1016/j.jfineco.2005.02.002.

Di Giovanni, Julian, Andrei A. Levchenko, and Isabelle Mejean. 2014. "Firms, Destinations, and Aggregate Fluctuations." *Econometrica* 82 (4): 1303–40. doi:10.3982/ECTA11041.

Dixit, Avinash. 1989. "Entry and Exit Decisions under Uncertainty." *Journal of Political Economy* 97 (3): 620. doi:10.1086/261619.

Donoghue, Ted O, and Jason Somerville. 2018. "Modeling Risk Aversion in Economics." *Journal of Economic Perspectives* 32 (2): 91–114. doi:10.1257/jep.32.2.91.

Easley, David, and Jon Kleinberg. 2010. *Networks , Crowds , and Markets : Reasoning about a Highly Connected World*. *Science*. Vol. 81. doi:10.1017/CBO9780511761942.

Facchinei, Francisco, and Christian Kanzow. 2010. "Generalized Nash Equilibrium Problems." *Annals of Operations Research* 175 (1): 177–211. doi:10.1007/s10479-009-0653-x.

Foerster, Andrew T, Pierre-daniel G Sarte, and Mark W Watson. 2011. "Sectoral versus Aggregate Shocks: A Structural Factor Analysis of Industrial Production." *Journal of Political Economy* 119 (1): 1–38. doi:10.1086/659311.

Fortin, Bernard, and Guy Lacroix. 2007. "Tax Evasion and Social Interactions." *Journal of Public Economics* 91 (11/12): 2089–2112. doi:10.1016/j.jpubeco.2007.03.005.

Gabaix, Xavier. 2011. "The Granular Origins of Aggregate Fluctuations." *Econometrica* 79 (3): 733–772. doi:10.3982/ECTA9623.

———. 2016. "Power Laws in Economics: An Introduction." *Journal of Economic Perspectives* 30 (1): 185–206. doi:10.1257/jep.30.1.185.

Gaertner, Fabio B. 2014. "CEO After-Tax Compensation Incentives and Corporate Tax Avoidance." *Contemporary Accounting Research* 31 (4): 1077–1102. doi:10.1111/1911-3846.12058.

Gordon, Roger, and Wei Li. 2009. "Tax Structures in Developing Countries: Many Puzzles and a Possible Explanation." *Journal of Public Economics* 93 (7–8). Elsevier B.V.: 855–66. doi:10.1016/j.jpubeco.2009.04.001.

Goyal, Sanjeev. 2016. *Networks in Economics*. Edited by Yann Bramoullé, Andrea Galeotti, and Brian Rogers. *The Oxford Handbook of the Economics of Networks*. Vol. 1. Oxford University Press. doi:10.1093/oxfordhb/9780199948277.013.3.

Grassi, Basile. 2017. "IO in I-O: Size, Industrial Organization, and the Input-Output Network Make a Firm Structurally Important." *Working Paper*.

Hallsworth, Michael. 2014. "The Use of Field Experiments to Increase Tax Compliance." *Oxford Review of Economic Policy* 30 (4): 658–79. doi:10.1093/oxrep/gru034.

Hanlon, Michelle, and Shane Heitzman. 2009. "A Review of Tax Research." *SSRN ELibrary*. http://ssrn.com/paper=1476561.

Harju, Jarkko, Tuomas Kosonen, and Olli Ropponen. 2014. “Do Honest Hairdressers Get a Haircut ? On Tax Rate and Tax Evasion.”

Hasseldine, John, Peggy Hite, Simon James, and Marika Toumi. 2007. “Persuasive Communications: Tax Compliance Enforcement Strategies for Sole Proprietors.” *Contemporary Accounting Research* 24 (1): 171–94. doi:10.1506/P207-004L-4205-7NX0.

Iyer, Govind S, Philip M J Reckers, and Debra L Sanders. 2010. “Increasing Tax Compliance in Washington State: A Field Experiment.” *National Tax Journal* 63 (1): 7–32.

Jackson, Matthew O. 2008. *Social and Economic Networks*. *Network*.

———. 2009. “An Overview of Social Networks and Economic Applications.” In *The Handbook of Social Economics*, 1–96.

———. 2014. “The Past and Future of Network Analysis in Economics.”

Jiménez, JP, JC Gómez Sabaini, and A Podestá. 2010. *Evasión y Equidad En América Latina*.

Jones, Charles I. 2013. “Misallocation, Economic Growth, and Input-Output Economics.” *Advances in Economics and Econometrics,Tenth World Congress* II: 419–54. doi:10.3386/w16742.

Kleven, Henrik Jacobsen, Martin B Knudsen, Claus Thustrup Kreiner, S\oren S\oren Pedersen, and Emmanuel Saez. 2011. “Unwilling or Unable to Cheat? Evidence From a Tax Audit Experiment in Denmark.” *Econometrica* 79 (3): 651–92. doi:10.2307/41237767.

Kleven, Henrik Jacobsen, Claus Thustrup Kreiner, and Emmanuel Saez. 2016. “Why Can Modern Governments Tax So Much? An Agency Model of Firms as Fiscal Intermediaries.” *Economica* 83 (330). Cambridge, MA: 219–46. doi:10.1111/ecca.12182.

Kopczuk, Wojciech, and Joel Slemrod. 2006. “Putting Firms into Optimal Tax Theory.” *American Economic Review* 96 (2): 130–34. doi:10.1257/000282806777212585.

Kosonen, Tuomas, and Olli Ropponen. 2015. “The Role of Information in Tax Compliance: Evidence from a Natural Field Experiment.” *Economics Letters* 129. Elsevier B.V.: 18–21. doi:10.1016/j.econlet.2015.01.026.

Kreutzer, David, and Dwight R Lee. 1988. “Tax Evasion and Monopoly Output Decisions: A Reply.” *National Tax Journal* 41 (1): 583–85.

Kwon, H Dharma. 2010. “Invest or Exit? Optimal Decisions in the Face of a Declining Profit Stream.” *Operations Research* 58 (3): 638–49. doi:10.1287/opre.1090.0740.

Lambrecht, Bart M. 2001. “The Impact of Debt Financing on Entry and Exit in a Duopoly.” *Review of Financial Studies* 14 (3): 765–804. doi:10.1093/rfs/14.3.765.

Levitt, Steven D, and John A List. 2007. “What Do Laboratory Experiments Measuring Social

Preferences Reveal About the Real World?” *Journal of Economic Perspectives* 21 (2): 153–74. doi:10.1257/jep.21.2.153.

Long, John B., and Charles I. Plosser. 1983. “Real Business Cycles.” *Journal of Political Economy* 91 (1): 39–69. doi:10.1086/261128.

Lorenzen, Gunter. 1990. “A Unified Approach to the Calculation of Growth Rates.” *The American Statistician* 44 (2): 148. doi:10.2307/2684156.

Lucas, Robert E. 1977. “Understanding Business Cycles.” *Carnegie-Rochester Conference Series on Public Policy, Elsevier* 5 (1): 7–29. doi:10.1016/0167-2231(77)90002-1.

Marrelli, Massimo. 1984. “On Indirect Tax Evasion.” *Journal of Public Economics* 25 (1–2): 181–96. doi:10.1016/0047-2727(84)90050-1.

Mas-Colell, Andreu. 1999. “The Future of General Equilibrium.” *Spanish Economic Review* 1 (3): 207–14.

Mcfadden, Daniel L. 1976. “Quantal Choice Analaysis : A Survey.” *Annals of Economic and Social Measurement* 5 (4).

Mcgraw, Kathleen M, John T Scholz, Kathleen M Mcgraw, and John T Scholz. 1991. “Appeals to Civic Virtue versus Attention to Self-Interest : Effects on Tax Compliance.” *Law & Society Review* 25 (3): 471–98.

Murphy, Richard, and John Christensen. 2013. “Tax Us If You Can.” *Tax Justice Network*, no. 2nd: 68.

Myles, Gareth D. 2001. *Public Economics*. Edited by Cambridge University Press. http://people.exeter.ac.uk/gdmyles/papers/pdfs/pubec.pdf.

Phillips, John D. 2003. “Corporate Tax-Planning Effectiveness: The Role of Compensation-Based Incentives.” *Accounting Review* 78 (3): 847–74. doi:10.2307/3203228.

Pomeranz, Dina. 2013. “No Taxation without Information: Deterrence and Self-Enforcement in the Value Added Tax.” *The American Economic Review* w19199 (8): 2539–69. doi:10.1257/aer.20130393.

Powers, Kathleen, John R. Robinson, and Bridget Stomberg. 2016. *How Do CEO Incentives Affect Corporate Tax Planning and Financial Reporting of Income Taxes? Review of Accounting Studies*. Vol. 21. Springer US. doi:10.1007/s11142-016-9350-6.

Robben, Henry S J, Paul Webley, Henk Elffers, and Dick J. Hessing. 1990. “Decision Frames, Opportunity and Tax Evasion. An Experimental Approach.” *Journal of Economic Behavior and Organization* 14 (3): 353–61. doi:10.1016/0167-2681(90)90063-J.

Sandmo, Agnar. 1971. “On the Theory Under of Price the Competitive Uncertainty Firm.” *The American Economic Review* 61 (1): 65–73.

———. 2005. “The Theory of Tax Evasion: A Retrospective View.” *National Tax Journal* 58 (4): 643–63.

Slemrod, Joel. 2004. “The Economics of Corporate Tax Selfishness.” *National Tax Journal* LVII (4): 877–99.

Slemrod, Joel, M Blumenthal, and C Christian. 2001. “Taxpayer Response to an Increased Probability of Audit: Results from a Controlledexperiement in Minnesota.” *Journal of Public Economics* 79: 455–83.

Slemrod, Joel, and Shlomo Yitzhaki. 2002. “Chapter 22 Tax Avoidance, Evasion, and Administration.” *Handbook of Public Economics* 3: 1423–70.

Stella, Andrea. 2015. “Firm Dynamics and the Origins of Aggregate Fluctuations.” *Journal of Economic Dynamics and Control* 55 (1133): 71–88. doi:10.1016/j.jedc.2015.03.009.

Tax Justice Network. 2007. “Closing the Floodgates. Collecting Tax to Pay for Development.” *Tax Justice Network*.

Torgler, Benno. 2007. *Tax Compliance and Tax Morale : A Theoretical and Empirical Analysis*. Cheltenham, UK ; Northampton, MA: Edward Elgar. http://eprints.qut.edu.au/15324/.

Tornqvist, Leo, Pentti Vartia, and Yrjo O. Vartia. 1985. “How Should Relative Changes Be Measured?” *The American Statistician* 39 (1): 43–46. doi:10.1080/00031305.1985.10479385.

Virmani, Arvind. 1989. “Indirect Tax Evasion and Production Efficiency.” *Journal of Public Economics* 39 (2): 223–37. doi:10.1016/0047-2727(89)90041-8.

von Heusinger, Anna. 2009. “Numerical Methods for the Solution of the Generalized Nash Equilibrium Problem.”